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Mathematical capture of failure processes in elastoplastic geomaterials

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Abstract

This paper discusses a strategy to identify failure conditions in geomaterials simulated by elastoplastic constitutive laws. The main objective is to express different forms of failure through the same formalism. For this purpose, we use a set of material instability indices combining the concepts of loss of controllability and critical hardening modulus with a simple, but versatile, elastoplastic model for soils and soft rocks. This choice has allowed us to (i) compute the instability indices in analytical form, (ii) capture the implications of non-normality and prior deposition/lithification history and (iii) inspect a broad range of failure modes (e.g., brittle and ductile failure, static liquefaction and compaction banding). It is shown that, although each mode of failure has its own specific features, they can all be encapsulated in a unified mathematical representation. To obtain these results, the instability moduli must reflect the static/kinematic constraints that generate the failure process at stake. Thus, the instability indices are expressed as functions of both the hardening modulus and additional terms of kinematic origin, with the latter terms reflecting a control-dependence of the plastic response. Such results describe a procedure for achieving a unified definition of failure in elastoplastic geomaterials, which is closely linked to the theory of controllability and encompasses the intuitive notions of 'hardening' and 'softening' as particular cases.

Keywords: Geomaterials; Elastoplasticity; Failure; Liquefaction; Compaction bands

1. Introduction

The mechanical response of geomaterials is affected by various factors such as the geologic history, the microstructure and the interaction with pore fluids (Leroueil and Vaughan, 1990; Gens, 2010). These factors cause incremental non-linearity, path-dependence and strength properties that vary with the stress history, drainage conditions and deformation paths (Tamagnini and Viggiani, 2002; Darve and Nicot., 2005). Constitutive laws for engineering applications can reproduce such features only via sophisticated mathematical strategies. An example is critical state plasticity (Muir Wood, 1990), which has enriched existing theories for plastic continua by linking the hardening/softening of soils to their plastic volumetric strains, thus expanding the range of failure modes that could be simulated by a single plastic model. Since then, constitutive laws have been enhanced to accommodate other features of soil behavior, such as non-normality (Nova and Wood, 1979), density-dependence (Gajo and Wood, 1999; Manzari and Dafalias, 1997), cementation and structure (Gens et al., 1993; Rouainia and Wood, 2000) and fabric anisotropy (di Prisco

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et al., 1993; Pestana and Whittle, 1999). A consequence of such improvements is that predictions of failure are no longer obvious; they may depend on the initial state and the imposed deformation patterns in ways that are not always intuitive.

This paper builds on a recently proposed theory for detecting the material instabilities in elastoplastic solids (Buscarnera et al., 2011) and addresses the problem of predicting geomaterial failure through a unified strategy that can be customized to multiple combinations of elastoplastic models, loading conditions and failure modes. This approach implements key ideas of similar theories (e.g., Maier and Hueckel, 1979; Bigoni and Hueckel, 1991; Imposimato and Nova, 1998; Nicot et al., 2007) and links them to the notion of a critical hardening modulus. As a result, it offers a simple strategy for investigating failure in both hardening and softening regimes, thus capturing instabilities that may take place at the transition from elastic to plastic states. In what follows, we discuss the application of this theoretical framework to the detection of different types of material instabilities. For this purpose, we have monitored various scalar indices of failure (here referred to as instability moduli) during numerical simulations. Simplicity has been considered as a key requirement, and analytical relations have been obtained to elucidate the source of the predicted failure mechanisms. For this reason, we have adopted a simple, but versatile, elastoplastic model that has allowed us to locate failure domains in the stress space. It is worth noting that the simulations discussed in the following, and the conclusions obtained from them, are specific to the selected model. As a result, their purpose is simply to elucidate the common mathematical roots of typical failure mechanisms of geomaterials, rather than discussing in quantitative terms the general features of their stress-strain-failure response. The study identifies three classes of geomaterials: (a) clays, (b) sands and (c) high-porosity rocks. Considering these model materials, the effect of key factors, such as preconsolidation history, non-normality and the degree of cementation, is discussed from an analytical standpoint. The following sections make reference to fluid-saturated porous solids and assume the validity of the effective stress principle. Matrix notation and compression-positive convention will be adopted. A dash indicates effective stresses (i.e., $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - u\boldsymbol{\delta}$, with $\boldsymbol{\sigma}$ and u being total stress and pore pressure, respectively, and $\boldsymbol{\delta}$ being the vector form of Kronecker's delta), while superposed T indicates transpose.

2. Mathematical capture of failure in elastoplastic solids

This section summarizes the theory proposed by Buscarnera et al. (2011), specializing it to triaxial stress conditions. The equations derived here will be used in the subsequent numerical simulations.

2.1. Material failure as a loss of the existence and/or uniqueness of the incremental plastic solution

Failure occurs in experiments if a sample material cannot sustain a specific form of incremental loading. Traditionally, the characterization of failure in solids involves the assessment of envelopes in the stress space representing the locus where failure has occurred. According to this approach, if the stress path is within this envelope, the material can sustain any incremental loading path. Such a classical view, however, does not hold up for geological materials because of the existence of inelastic phenomena within this envelope that can give rise to failure. These phenomena, hereafter referred to as material instabilities, are characterized by the fact that it is possible to generate a passage from a quasi-static to a dynamic regime of deformation even with no external energy input (Nicot et al., 2007). In saturated sands, for example, an impressive form of instability is the liquefaction of loose sands (Lade, 1992; Borja, 2006). In the broader domain of geomechanics, unexpected instabilities are possible even under highly constrained conditions, as in the case of the localized compaction bands of porous rocks subjected to one-dimensional compression (Arroyo et al., 2005). The above-mentioned instabilities occur at stress levels that do not correspond to frictional failure domains and that can be overlooked by classical approaches (Nova et al., 2003). As a result, since the region inside the failure envelope is no longer a guarantee of material stability, classical strategies must be replaced by more sophisticated approaches. The approach pursued here involves the definition, in analytical form, of domains at which the existence and/or uniqueness of the incremental solution at material point levels is not guaranteed for an imposed set of control conditions.

The identification of these domains is embedded in the expression for the elastoplastic functions, such as the yield surface ($f(\boldsymbol{\sigma}', \boldsymbol{\Psi}) = 0$, where $\boldsymbol{\Psi}$ is a set of internal variables), the plastic potential ($g\left(\boldsymbol{\sigma}', \tilde{\boldsymbol{\Psi}}\right)$, where $\tilde{\boldsymbol{\Psi}}$ is a set of dummy variables) and the hardening rules ($\boldsymbol{\Psi} = \boldsymbol{\Psi}(\boldsymbol{\varepsilon}^{\mathbf{p}})$). In particular, the stress state and its increments are bounded to satisfy the following constraints:

$$f \le 0, \quad f\Lambda = 0 \quad \text{and} \quad \Lambda \ge 0 \tag{1}$$

where Λ is a non-negative plastic multiplier. Violations of the conditions in Eq. (1) are a source of the potential loss of the uniqueness and/or existence of the incremental plastic solution. This aspect has long been recognized by the early concept of controllability (Imposimato and Nova, 1998), that relates the existence/uniqueness of the elastoplastic solution to the imposed set of static/kinematic conditions. Buscarnera et al. (2011) have recently showed that the interplay between the selected loading program and the existence/uniqueness of the incremental solution can also be disclosed by a proper rearrangement of the consistency requirement. Under very general control conditions, it reads as

$$\frac{\partial f}{\partial \boldsymbol{\sigma}'_{\alpha}} \dot{\boldsymbol{\sigma}}'_{\alpha} + \frac{\partial f}{\partial \boldsymbol{\sigma}'_{\beta}} \left[\boldsymbol{D}^{e}_{\beta\alpha} (\boldsymbol{D}^{e}_{\alpha\alpha})^{-1} \dot{\boldsymbol{\sigma}}'_{\alpha} + \left(\boldsymbol{D}^{e}_{\beta\beta} - \boldsymbol{D}^{e}_{\beta\alpha} (\boldsymbol{D}^{e}_{\alpha\alpha})^{-1} \boldsymbol{D}^{e}_{\alpha\beta} \right) \dot{\boldsymbol{\varepsilon}}_{\beta} \right] - (H - H_{\chi})\Lambda = 0 \tag{2}$$

where H is the hardening modulus, H_{χ} is a control-dependent term (controllability modulus), $\dot{\boldsymbol{\phi}} = \left[\dot{\boldsymbol{\sigma}}'_{\boldsymbol{\alpha}}, \dot{\boldsymbol{\varepsilon}}_{\boldsymbol{\beta}} \right]$ is a control variable, $\dot{\boldsymbol{\psi}} = \left[\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{\alpha}}, \dot{\boldsymbol{\sigma}}'_{\boldsymbol{\beta}} \right]$ is a response variable, and $\mathbf{D}^{e}_{ij}$ are partitions of the elastic stiffness matrix. The controllability moduli can also be expressed as a function of a partition of the elastic compliance