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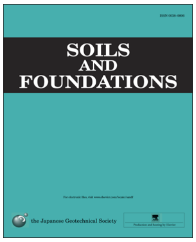

Discussion on size effect of footing in ultimate bearing capacity of sandy soil using rigid plastic finite element method

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Abstract

Currently, many formulas are used to calculate the ultimate bearing capacity. However, these formulas have disadvantages when being applied in practice since they can only be applied for calculating simple footing shapes and uniform grounds. Most formulas do not take into account the size effect of the footing on the ultimate bearing capacity, except for the formula by the Architectural Institute of Japan. The advantage of using the finite element method (FEM) is its applicability to non-uniform grounds, for example, multi-layered and improved grounds, and to complicated footing shapes under three-dimensional conditions. FEM greatly improves the accuracy in estimating the ultimate bearing capacity. The objective of this study is to propose a rigid plastic constitutive equation using the non-linear shear strength property against the confining pressure. The constitutive equation was built based on experiments for the non-linear shear strength property against the confining pressure reported by Tatsuoka and other researchers. The results from tests on Toyoura sand and various other kinds of sand indicated that, although the internal friction angle differs among sandy soils, the normalized internal friction angle decreases with an increase in the normalized first stress invariant for various sands despite dispersion in the data. This property always holds irrespective of the reference value of the confining pressure in the normalization of the internal friction angle. The applicability of the proposed rigid plastic equation was proved by comparing it to the ultimate bearing capacity formula by the Architectural Institute of Japan, which is an experimental formula that takes into account the size effect of the footing. The results of rigid plastic finite element method (RPFEM) with the proposed constitutive equation were found to be similar to those obtained with the Architectural Institute of Japan's formula. It is clear that RPFEM, with the use of the non-linear shear strength against the confining pressure, provides good estimations of the ultimate bearing capacity of the footing by taking account of the size effect of the footing.

Keywords: Ultimate bearing capacity; Size effect; Stress-dependent shear strength; Finite element method

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1. Introduction

In the design of buildings, the assessment of the ultimate bearing capacity of the footing is an important task in order to examine the stability of the building-ground system. Pioneering works were conducted by Prandtl (1921) and Reissner (1924). Prandtl considered a rigid-perfectly plastic half space loaded by a strip punch. The punch–soil interface can be frictional or smooth, and the material is set as weightless. The stress boundary condition is zero traction on the surface of the half space, except for the strip punch. Prandtl proposed bearing capacity factor N_c by analytical consideration. Reissner (1924) analyzed a similar problem, but there are two conditions different from those of Prandtl. The material is set as purely frictional ($c=0$), and a uniformly distributed pressure is loaded at the surface of the half space. Reissner applied hyperbolic-type equations to solve the boundary value problem and introduced bearing capacity factor N_q. In the case of frictional-cohesive material, the analyzed slip-line is obtained similarly to the slip-line field. Bearing capacity factors N_q and N_c are adopted for many ultimate bearing capacity formulae. The ultimate bearing capacity formula for the footing by Terzaghi (1943) has been widely employed in practice. It takes into account the effects of cohesion, surcharge and soil weight (Terzaghi, 1943). The ultimate bearing capacity formula is typically expressed as

$$q = cN_c + 1/2\gamma B N_\gamma + \gamma D_f N_q \tag{1}$$

where N_c, N_γ and N_q are the bearing capacity factors, which are functions of the internal friction angle of the soil, ϕ. The other indexes are as follows:

γ : unit weight of soil (kN/m^3),
D_f : depth of footing (m), and
B : footing width (m)

Since this approach has been proposed, various studies on bearing capacity factors have been conducted. Bearing capacity factors N_q and N_c were provided by Prandtl (1921) and Reissner (1924) as

$$N_q = e^{\pi \tan\phi} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \tag{2}$$

$$N_c = \left(N_q - 1\right)\cot\phi \tag{3}$$

With regard to the N_γ factor, several formulations have been proposed, but no formula is totally accurate. For example, the formula by Meyerhof (1963) is expressed in the following way:

$$N_\gamma = \left(N_q - 1\right)\tan(1.4\phi) \tag{4}$$

Meyerhof (1951, 1963) introduced other factors, such as semi-empirical inclination factors i_c, i_γ and i_q. The ultimate bearing capacity formula is described as follows:

$$q = i_c c N_c + 1/2 i_\gamma \gamma_1 B N_\gamma + i_q \gamma_2 D_f N_q \tag{5}$$

$$i_c = i_q = \left(1 - \frac{\theta}{90^\circ}\right)^2 \tag{6}$$

$$i_\gamma = \left(1 - \frac{\theta}{\phi}\right)^2 \tag{7}$$

where θ is the inclination angle of the load with respect to the vertical plane.

The Architectural Institute of Japan (AIJ, 1988, 2001) developed the ultimate bearing capacity formula and it is now widely used in Japan. It was developed semi-experimentally. By using factors N_c and N_q , given by Prandtl, and N_γ, described by Meyerhof, the ultimate bearing capacity formula is expressed as follows:

$$q = i_c \alpha c N_c + i_\gamma \gamma_1 \beta B \eta N_\gamma + i_q \gamma_2 D_f N_q \tag{8}$$

In the above equation, α and β express shape coefficients for which $\alpha = 1$ and $\beta = 0.5$ are recommended by De Beer (1970). η is the size effect factor defined in the following:

$$\eta = \left(\frac{B}{B_0}\right)^m \tag{9}$$

where B_0 is the reference value in the footing width, m is the coefficient determined from the experiment and $m = -1/3$ is recommended in practice.

The ultimate bearing capacity formula by AIJ successfully takes into account the size effect of the footing which has not been considered in past formulae employing the Mohr–Coulomb criteria for soil strength. Since the past formulae overestimated the ultimate bearing capacity with the increase in footing width, this effect needs to be examined. Ueno et al. (1998) reported that the size effect on the ultimate bearing capacity was mainly attributed to the stress level effect on the shear strength of soils. Their research indicated that the mean stress ranged from $2\gamma B$ to $10\gamma B$ beneath the footing and caused changes in the internal friction angle of the ground mainly due to the mean stress. This study attempts to discuss the size effect on the ultimate bearing capacity by using a finite element analysis with the rigid plastic constitutive equation, which simulates the non-linear shear strength property of sandy soil against the confining pressure.

In recent years, the finite element method (FEM) has become widely accepted as one of the well-established and convenient techniques for solving complex problems in various fields of engineering and mathematical physics. The latest four decades have observed a growing use of the finite element method in geotechnical engineering. FEM has been applied to estimate the bearing capacity of strip footings on cohesionless soils, such as Sloan and Randolph (1982), Griffiths (1982) and Frydman and Burd (1997). The rigid-plastic finite element method (RPFEM) was developed for geotechnical engineering by Tamura et al. (1984) and Tamura et al. (1987a, 1987b). In this method, the limit load is calculated without the assumption on the potential failure mode. The method is effective in calculating the ultimate bearing capacity of a footing against three-dimensional boundary value problems where the soil conditions are varied as a multi-layered ground. Although RPFEM was originally developed based on the upper bound theorem in plasticity, Tamura et al. proved that it could be derived directly using the

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