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Analytical layer-element solution for 3D transversely isotropic multilayered foundation

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Abstract

This paper presents a solution for 3D transversely isotropic multilayered foundations under external loading, for which the analytical layer-element method is used because it has good numerical stability due to its symmetry and no existence of positive exponents. Based on the basic equations for transversely isotropic elastic materials, an analytical layer-element, which describes the relationship between the displacements and stresses of 3D transversely isotropic single-layered foundations, is exactly derived in the transformed domain by applying the double Fourier transform technique and the Cayley–Hamilton theorem. Taking account of the continuity conditions between adjacent layers, the global stiffness matrix can be obtained by assembling the interrelated layer elements. The solutions in the physical domain can be obtained by a numerical Fourier inversion. Finally, numerical examples are carried out to verify the presented theory and to elucidate the effect of stratification on the deformation of a foundation.

Keywords: Analytical layer-element; 3D transverse isotropy; Multilayered foundation; Fourier transform

1. Introduction

The deformation response of soil due to external loading is an important issue in the design of foundations. For simplification, the estimations of displacements and stresses in materials are presently based more on models that consider the materials as homogeneous, linearly elastic, and isotropic media. As is known, in most cases, natural soils have different mechanical properties in different directions since most of them are deposited through a geological process of sedimentation that occurs over a long period of time. Therefore, it is more reasonable to use a multilayered transversely isotropic model, than an isotropic model, to describe the mechanical character of a foundation.

As the point force solution is the origin of more complex loading problems, many investigators have used different methods to obtain such solutions for transversely isotropic bodies in different cases. Barden (1963) obtained the concentrated force solutions in a half-space by extending the corresponding approaches for isotropic media, which can calculate vertical surface displacements and stresses on the load axis. In the same way, Pan (1989a) obtained a solution in three-dimensional infinite space. By employing Green's function, Pan and Chou (1976, 1979) provided closed-form fundamental solutions for the elastic responses of a transversely isotropic solid of either infinite extent or semi-infinite extent subjected to concentrated point forces. Considering the effect of body force, Chowdhury (1987) created a solution to an axisymmetric boundary value problem of

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a semi-space of transversely isotropic material whose interior was applied by a point force via images and Hankel transforms. In addition, Yue (1995) presented closed-form fundamental solutions for elastic fields in two joined dissimilar transversely isotropic semi-infinite solids due to concentered forces. Liao et al. (1998) proposed complete closed-form solutions for displacements and stresses subjected to a point load in a transversely isotropic elastic half-space by superimposing the solutions of two infinite spaces, one having a point load in its interior and the other subjected to free loading. Karapetian and Kalinin (2011) also presented an explicit solution for the fields of displacement induced by a concentrated point source applied to the surface of a three-dimensional semi-infinite transversely isotropic solid according to the potential theory.

Work concerning the induced displacement in transversely isotropic media by other loading types has been done by some investigators. By extending the point load solutions (Liao et al., 1998), Wang and Liao (1999) obtained elastic closed-form solutions for displacements and stresses in a transversely isotropic half-space subjected to various buried loading types which include finite line loads and asymmetric loads (such as uniform and linearly varying rectangular loads or trapezoidal loads). Furthermore, Wang and Tzeng (2009) presented solutions for displacements and stress components along the centerline of a non-uniform circular distribution of vertical loads in a continuously inhomogeneous cross-anisotropic material with Young's and shear moduli varying exponentially with depth by integrating the point load solutions in a cylindrical co-ordinate system for an inhomogeneous cross-anisotropic half-space, which were derived by Wang et al. (2003). Utilizing Fourier transforms and the variation in parameters, Wang et al. (2010) also derived fundamental solutions for horizontal and vertical line loads acting in a continuously inhomogeneous plane strain cross-anisotropic full space. Using the classical Fourier integral transforms, Yue et al. (2005) derived the closed-form solutions for the complete elastic field in a semi-infinite solid subjected to distributed rectangular loading. To the best of the authors' knowledge, a majority of the above-mentioned theories have not been applied to layered foundations. With the technique of double Fourier transforms, Hankel transforms and the finite layer method, Small and Booker (1984, 1986) obtained expressions for deformations in two- and three-dimensions for a transversely isotropic and layered material by surface loads in terms of flexibility matrix, which overcomes the difficulty that a finite layer stiffness approach breaks down when applied to incompressible materials. Singh (1986) and Garg and Singh (1987) used the method of propagator matrix to solve the problems of the axially-symmetric deformation and two-dimensional deformation, respectively, of a transversely isotropic multilayered half-space by surface loads. Utilizing the same method and introducing two systems of vector functions, Pan (1989b, 1989c) obtained a solution for a transversely isotropic and layered elastic half-space under the action of general surface loads or by general dislocation sources.

Usually, the elements in the propagator matrix have positive exponential functions, which may result in unstable numerical results under the influence of numerically ill-conditioned matrices and computation overflow. Recently, the analytical layer-element method has been developed to analyze the multilayered transversely isotropic elastic media subjected to axisymmetric and non-axisymmetric loading (Ai et al., 2012; Ai and Cheng, 2013), which has good numerical stability for its symmetry and no existence of positive exponents. Noting that rectangular loading may occur frequently in the engineering field and that the analytical layer-element method has the above advantages, we aim to extend this method in the present study by including such loading in a three-dimensional analysis. Starting from the basic equations for transversely isotropic elastic materials, the analytical layer-element in the transformed domain is obtained by the application of the state vector method, the technique of a double Fourier transform, and the Cayley–Hamilton theorem. The global stiffness matrix is obtained by assembling the interrelated layer elements according to the continuity conditions between adjacent layers. The actual solutions in the physical domain are recovered by the numerical inversion of the double Fourier transform. In the end, the solutions presented in this study are verified and some illustrative examples are given to investigate the effect of the material properties of transversely isotropic and multilayer media.

2. Analytical layer-element for a single material layer

In a 3-dimensional Cartesian coordinate system, the stress equilibrium equation in the absence of body forces for the elastic body can be expressed as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \tag{1a}$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \tag{1b}$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \tag{1c}$$

in which σ_x, σ_y, and σ_z are the normal stress components in the x, y, and z directions, respectively; τ_{yz}, τ_{xz}, and τ_{xy} are the shear stress components in the yz, xz, and xz planes, respectively.

The constitutive equations of the transversely isotropic elastic body, in terms of the displacements, are expressed as

$$\sigma_x = c_{11}\frac{\partial u_x}{\partial x} + c_{12}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z} \tag{2a}$$

$$\sigma_y = c_{12}\frac{\partial u_x}{\partial x} + c_{11}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z} \tag{2b}$$

$$\sigma_z = c_{13}\frac{\partial u_x}{\partial x} + c_{13}\frac{\partial u_y}{\partial y} + c_{33}\frac{\partial u_z}{\partial z} \tag{2c}$$

$$\tau_{yz} = c_{44}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \tag{2d}$$

$$\tau_{xz} = c_{44}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \tag{2e}$$