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The inclination and shape factors for the bearing capacity of footings ☆

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Abstract

In 1920, Prandtl published an analytical solution for the bearing capacity of a maximum strip load on a weightless infinite half-space. Prandtl subdivided the sliding soil component into three zones: two triangular zones on the edges and a wedge-shaped zone in between the triangular zones that has a logarithmic spiral form. The solution was extended by Reissner (1924) with a surrounding surcharge. Nowadays, a more extended version of Prandtl's formula exists for the bearing capacity. This extended formulation has an additional bearing capacity coefficient for the soil weight and additional correction factors for inclined loads and non-infinite strip loads. This extended version is known in some countries as "The equation of Meyerhof", and in other countries as "The equation of Brinch Hansen", because both men have separately published solutions for these additional correction factors. In this paper, we numerically solve the stresses in the wedge zone and derive the corresponding bearing capacity coefficients and inclination and shape factors. The inclination factors are also analytically solved.

Keywords: Bearing capacity; Footings; Foundations; Inclination factor; Shape factor

1. Introduction

In 1920, the German engineer Ludwig Prandtl published an analytical solution for the bearing capacity of soil under a limit pressure, p, causing the kinematic failure of the weightless infinite half-space underneath. The strength of the half-space is given by the angle of the internal friction, ϕ, and the cohesion, c. The solution was extended by Reissner (1924) with a surrounding surcharge, q. Prandtl subdivided the sliding soil part into three zones (Fig. 1):

1. Zone 1: A triangular zone below the strip load with a width $B=2\cdot b_1$. Since there is no friction on the ground surface, the directions of the principal stresses are horizontal and vertical; the largest principal stress is in the vertical direction.
2. Zone 2: A wedge with the shape of a logarithmic spiral, where the principal stresses rotate 90° from Zone 1 to Zone 3. The pitch of the sliding surface equals the angle of internal friction ϕ, creating a smooth transition between Zone 1 and Zone 3 and also creating a zero frictional moment on this wedge (see Eq. (13)).
3. Zone 3: A triangular zone adjacent to the strip load. Since there is no friction on the surface of the ground, the directions of principal stress are horizontal and vertical with the vertical component having the smallest amplitude.

The interesting part of the solution is that all three zones are fully failing internally, according to the Mohr–Coulomb failure criterion, while the outer surfaces are simultaneously fully sliding, according to the Coulomb failure criterion. Only the latter criterion exists in the case of a Bishop slope stability calculation. The analytical solution for the bearing capacity of this three-zone

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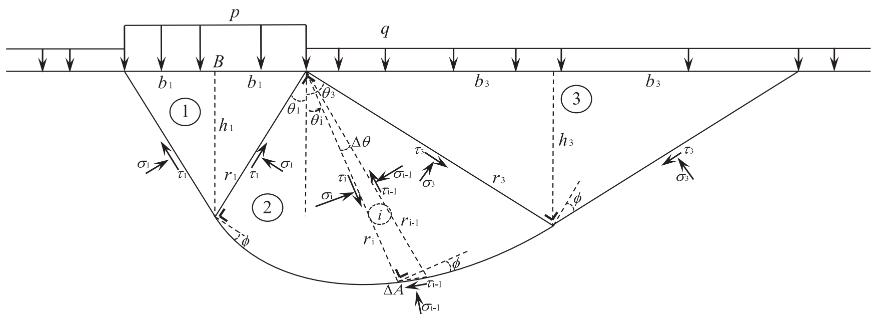

Fig. 1. Parameters used in the numerical approach to the wedge by Prandtl.

problem by Prandtl and Reissner can be written as

$$p = cN_c + qN_q \tag{1}$$

where the bearing capacity coefficients are given as

$$\begin{aligned} N_q &= K_p \cdot \exp(\pi \tan \phi) \\ N_c &= \left(N_q - 1\right)\cot \phi \end{aligned} \quad \text{with : } K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \tag{2}$$

This equation has been extended by Keverling Buisman (1940) for the soil weight, γ. Terzaghi (1943) wrote this extension as:

$$p = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma. \tag{3}$$

Keverling Buisman (1940); Terzaghi (1943); Meyerhof (1951, 1953, 1963); Caquot and Kérisel, 1953; Brinch Hansen (1970); Vesic (1973) and Chen (1975) subsequently proposed different equations for the soil weight-bearing capacity coefficient, N_γ. The equation by Brinch Hansen (note Brinch Hansen and not Hansen as presented in many texts), for the soil weight bearing capacity coefficient, was based on calculations of Lundgren-Mortensen and also of Odgaard and Christensen. The Chen equation for the soil weight-bearing capacity coefficient became the currently used equation

$$N_\gamma = 2\left(N_q - 1\right) \tan \phi. \tag{4}$$

This solution is rather close to the solution of Michalowski (1997) using the limit analyses and also the numerical results of Zhu and Michalowski (2005).

In 1953, Meyerhof was the first to propose equations for inclined loads. He was also the first, in 1963, to write the following formula for the vertical bearing capacity with both inclination factors and shape factors:

$$p_v = i_c s_c c N_c + i_q s_q q N_q + i_\gamma s_\gamma \frac{1}{2}\gamma BN_\gamma. \tag{5}$$

Further, he proposed equations for both the inclination factors and shape factors.

More recently, Brinch Hansen (1970) also wrote a formula for the bearing capacity like Eq. (5), but proposed other inclination and shape factors. This explains why in some countries Eq. (5) is known as "The equation of Meyerhof", and in other countries as "The equation of Brinch Hansen". In addition, in some countries, mainly in Asia, people work with the older "Equation of Terzaghi".

The inclination factors and shape factors of both Meyerhof and Brinch Hansen will be numerically evaluated in this paper.

2. Numerical approach for determining the bearing capacity coefficients

The three-zone problem of Prandtl can be solved using a numerical approach to determine the bearing capacity coefficient as a function of the angle of internal friction ϕ. The definitions of the parameters are shown in Fig. 1.

The Mohr–Coulomb failure criterion defines the angles in the triangular zones as

$$\theta_1 = \frac{1}{4}\pi - \frac{1}{2}\phi \ \text{ and } \ \theta_3 = \frac{1}{4}\pi + \frac{1}{2}\phi \ \text{ so } \ \theta_1 + \theta_3 = \frac{1}{2}\pi. \tag{6}$$

The length of both legs of the triangle can be determined from the width of the load strip ($B = 2 \cdot b_1$) and the size and shape of the logarithmic spiral, namely,

$$r(\theta) = r_1 \cdot \exp((\theta - \theta_1) \tan \phi) \tag{7}$$

giving

$$\frac{r_3}{r_1} = \exp\left(\frac{1}{2}\pi \tan \phi\right) \ \text{ and } \ \frac{b_3}{b_1} = \frac{r_3}{r_1} \tan \theta_3. \tag{8}$$

2.1. Zone 3

For Zone 3, the vertical stress is given by the surcharge ($\sigma_v = q = q_{\min}$) and the horizontal stress is given by the Mohr–Coulomb criterion as follows:

$$\sigma_h = \sigma_{\max} = \sigma_{\min} K_p + 2c\sqrt{K_p} \ \text{ with } K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \theta_3. \tag{9}$$

The normal stress, σ_3, is found using the principle of force equilibrium. The normal stresses are then split into different bearing components:

$$\begin{aligned} \frac{\sigma_{3,q}}{q} &= K_p \cdot \cos^2 \theta_3 + \sin^2 \theta_3 = 2 \sin^2 \theta_3, \\ \frac{\sigma_{3,c}}{c} &= 2\sqrt{K_p} \cdot \cos^2 \theta_3 = \cos \phi. \end{aligned} \tag{10}$$