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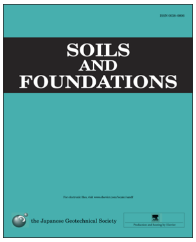

A stress–strain description of saturated sand under undrained cyclic torsional shear loading

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Abstract

A constitutive model to describe the cyclic undrained behavior of saturated sand is presented. The increments in volumetric strain during undrained loading, which are equal to zero, are assumed to consist of increments due to dilatancy and increments due to consolidation/swelling. This assumption enables the proposed model to evaluate increments in volumetric strain due to dilatancy as mirror images of increments in volumetric strain due to consolidation/swelling, thus simulating the generation of excess pore water pressure (i.e., reduction in mean effective principal stress) during undrained cyclic shear loading. Based on the results of drained tests, the increments in volumetric strain due to consolidation/swelling are evaluated by assuming that the quasi-elastic bulk modulus can be expressed as a unique function of the mean effective principal stress. On the other hand, in evaluating the increments in volumetric strain due to dilatancy, a normalized stress–plastic shear strain relationship is employed in combination with a novel empirical stress–dilatancy relationship derived for torsional shear loading. The proposed stress–dilatancy relationship accounts for the effects of over-consolidation during cyclic loading. Numerical simulations show that the proposed model can satisfactorily simulate the generation of excess pore water pressure and the stress–strain relationship of saturated Toyoura sand specimens subjected to undrained cyclic torsional shear loading. It is found that the liquefaction resistance of loose Toyoura sand specimens can be accurately predicted by the model, while the liquefaction resistance of dense Toyoura sand specimens may be slightly underestimated. (i.e., the liquefaction potential is higher). Yet, the model predictions are conservative.

Keywords: Liquefaction behavior; Stress–dilatancy relationship; Over-consolidation; Sand; Constitutive model

JEL classification: IGC; A10; D07; E08; T06

1. Introduction

Earlier experimental attempts to study the liquefaction behavior of soils date back to the 1960s when Seed and Lee (1966) conducted a series of undrained cyclic triaxial tests on saturated sand and reported that the onset of liquefaction was primarily governed by the relative density of the sand, the confining pressure, the stress or strain amplitude and the

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Nomenclature

$\tau_{z\theta}$	shear stress
σ'_z, σ'_r and σ'_θ	axial, radial and circumferential effective stress, respectively
p'	mean effective stress
$\mathrm{Dr}_{\mathrm{ini}}$	relative density measured at confining pressure of 30 kPa
$(\tau_{z\theta}/p')_{\max}$	maximum shear stress ratio
$\tau_{z\theta\ \max}$	peak shear stress
$G_{z\theta 0}$	initial quasi-elastic shear modulus $(=d\tau_{z\theta}/d\gamma^e_{z\theta})$
$\gamma_{z\theta}$, $\gamma^e_{z\theta}$, $\gamma^p_{z\theta}$	total, elastic and plastic shear strain, respectively (engineering strain)
$\varepsilon^p_{\mathrm{vol}}$	plastic volumetric strain
γ_{ref}	reference shear strain $(=(\tau_{z\theta}/p')/(G_{z\theta 0}/p'))$
m, n, k	material parameters that accounts for the stress induced anisotropy of Young's moduli, shear moduli and Poisson's ratio, respectively
C_E, C_G	factors that account for the degradation of quasi-elastic Young's and shear moduli, respectively (assumed as zero in the present study)
A	$E_{z0}/E_{\theta 0}$, i.e. ratio of vertical to circumferential quasi elastic Young's moduli at isotropic stress state
Y	normalized shear stress $(=(\tau_{z\theta}/p')/(\tau_{z\theta}/p')_{\max})$
X	normalized shear strain $(=\gamma^p_{z\theta}/\gamma_{\mathrm{ref}})$
D_1 and D_2	drag parameters
D	plastic shear moduli immediately after reversal of stress/initial plastic shear moduli (i.e., damage parameter)
D_{ult}	minimum value for D
S	amount of hardening
S_{ult}	maximum value for S
OC	over-consolidation ratio
$-d\varepsilon^p_{\mathrm{vol}}/d\gamma^p_{z\theta}$	dilatancy ratio
R_k	gradient of the empirical stress–dilatancy relationship
R_m	the maximum value for R_k
C	intercept of the empirical stress–dilatancy relationship
$C_{\min}$	minimum value for C

number of loading cycles. Since then, extensive studies have been conducted on soil liquefaction throughout the world (Vaid and Thomas, 1995, among others) and a number of attempts have been made to define proper constitutive models to describe it (Liou et al., 1977; Liyanapathirana and Poulos, 2002, among others).

Based on the results of several series of experiments on saturated hollow cylindrical sand specimens, Towhata and Ishihara (1985a) proposed a unique correlation between the shear work and the generation of pore water pressure (PWP). Furthermore, the effects of the rotation of the principal stress axes on sand liquefaction were investigated by Towhata and Ishihara (1985b) using hollow cylindrical specimens subjected to cyclic torsional shear loading. However, compared to the large amount of experimental data existing on liquefaction and the undrained behavior of soils, very few models are available to successfully simulate the soil performance under cyclic undrained loading. Ishihara et al. (1975) proposed a model based on five postulates to trace the generation of the excess PWP of sand subjected to undrained irregular cyclic loading. This model qualitatively simulates the stress–strain relationships and the shear stress versus mean effective stress relationships.

A constitutive model to simulate the cyclic undrained behavior of sand, based on the multi-spring concept, was developed by Iai et al. (1992). In this model, commonly known as the “Towhata–Iai model”, shear deformation is modeled by employing the multi-spring concept, and the generation of excess PWP is modeled using a unique correlation between the increments in excess PWP and shear work, as proposed by Towhata and Ishihara (1985a). Nishimura (2002) and Nishimura and Towhata (2004) modified the above model by expanding the multi-spring concept from two dimensions to three dimensions, while using an empirical stress–dilatancy relationship to model the generation of excess PWP by correlating the stress–dilatancy relationship to consolidation. Nevertheless, these models do not consider the inherent anisotropy of soils. Furthermore, the steady state during liquefaction and the continuous increase in shear strain with cyclic loading cannot be properly simulated.

An elasto-plastic constitutive model for sand, based on the non-linear kinematic hardening rule, was employed to investigate the effectiveness of the cement-mixing column method and the gravel drain method as countermeasures against liquefaction by means of a two-dimensional liquefaction analysis (Oka et al., 1992). Later, Oka et al. (1999) further modified this model by introducing a stress–dilatancy relationship that accounts for the damage to plastic stiffness at large levels of shear strain. In addition, several other constitutive models, based on the critical state framework, are proposed in the literature. Jefferies (1993) proposed a strain-hardening model, which utilizes the state parameter, to explain the behavior of very loose to very dense sand. A unified generalized plasticity model, based on the non-linear critical state line, was proposed by Ling and Yang (2006).

It should be noted that all the above-described models are based on either the critical state soil mechanics approach (e.g., Oka et al., 1992) or the energetic approach (Iai et al., 1992; Nishimura, 2002). In the current study a different and original approach is attempted by extending empirical relationships that are found to be reasonably consistent with the experimental observations. The undrained cyclic behavior of sand is simulated based on the response whereby the same sand is shown during drained cyclic loading. In fact, after appropriate normalization, the stress–strain relationship is found to be unique for drained and undrained conditions. Moreover, the generation of PWP during undrained loading can be described based on the volumetric strain response of sand during drained loading. This is done by improving the model proposed by De Silva and Koseki (2012) that can