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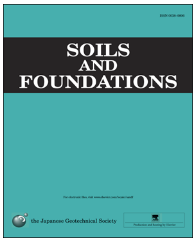

A modified solution of radial subgrade modulus for a circular tunnel in elastic ground

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Abstract

In models based on Winkler springs for tunnel lining design, designers always face the difficulty of selecting appropriate values for the radial subgrade modulus (k_r). The widely used solution k_r for a circular tunnel in elastic ground proposed by Wood (1975) was found to be applicable only when the tunnel radial deformation is oval-shaped. On the basis of the Wood's solution, this note presents a general solution for k_r when the radial deformation of the tunnel is described by a Fourier series. This modified Wood's solution of k_r using compatible stress functions is validated by a numerical example. The modified solution for the example shows good consistency with the original Wood's solution when the tunnel becomes an oval shape with deformations. The example indicates that the magnitude of k_r is significantly affected by the distribution shape of the tunnel radial deformation. The value of k_r is no longer a constant value around the tunnel when the tunnel deforms into a general shape described by a Fourier series. It is quite different from the value of k_r for a distribution shape described by a single Fourier term, i.e. one involving a single frequency. The application of a general solution for k_r is illustrated by a design case using a bedded beam model.

Keywords: Tunnel lining; Winkler model; Radial subgrade modulus; Fourier series; Elastic analysis

1. Introduction

Recommended by guidelines for the structural design of tunnel linings, the bedded beam model based on Winkler springs is widely used by tunnel design engineers (RTRI, 1997; ITA, 2004; JSCE, 2007). In this model, however, it is not easy for engineers to determine the radial subgrade modulus (k_r) appropriately (Duddeck and Erdmann, 1982; Mair, 2008; Gruebl, 2012). Table 1 shows values for k_r recommended by Standards in China and Japan for shield-driven tunnels. Table 1indicates that even when the soil is of a specific type, engineers still have to select a value from

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Nomenclature

A_m, B_m, C_m, D_m	coefficients in stress function
a_m	weight of the k_{rm}
E_C, v	soil Young's modulus, Poisson's ratio
k_r, k_{rm}	radial subgrade modulus, the value for a general mode m
m	shape mode of the tunnel radial deformation
u_r, u_θ	soil radial and tangential displacement
u_m	weight of a general mode m in Fourier series
r, r_0, r_1	radial coordinate, tunnel radius, radius of the large circle in FEM mesh
σ_r	soil radial stress changes
θ	angle measured counterclockwise from the tunnel crown

the wide range of k_r based on their own experience. Unfortunately, the structural behavior of the segmental lining and the joints have been found to be quite sensitive to the selected magnitude of k_r (Lee et al., 2001). Hence, some analytical solutions of k_r have been put forward as a rational method to determine the magnitude of this parameter (Arnau and Molins, 2011).

To derive these analytical solutions for k_r, the distribution shape of the radial deformation of the tunnel (hereafter referred to as the distribution shape) must be prescribed. Usually this distribution shape is assumed to be either circular (Sagaseta, 1987) or oval (Wood, 1975). However, a single distribution shape might not be sufficient to describe the actual behavior of the tunnel lining due to the complex soil-lining interactions. Hence, a solution of k_r based on the single circular or oval shape would be ideal for tunnel designs. In this note, an analytical solution for k_r for a more general distribution shape described by a Fourier series (Eq. (1)) is presented.

$$u_r = \sum_m u_m \cos m\theta \tag{1}$$

Besides the solution for an oval shape, Wood (1975) also presented a solution of k_r for the distribution shape described by a general term of the above Fourier series. However, the Airy stress function used in the Wood (1975)'s solution of k_r for the general term does not satisfy the strain compatibility equation identically. Wood (1975)'s solution is thus revised using compatible Airy stress functions to derive a complete solution of k_r for a general distribution shape described in the form of a Fourier series. A numerical example presented by Bobet (2001) is adopted to validate the proposed analytical solution of k_r. Finally, a design case is introduced to illustrate the applicability of the proposed solution of k_r to the bedded beam model.

2. Problem statement

Fig. 1 shows the problem geometry. A circular tunnel with radius r_0 is embedded in a homogeneous isotropic infinite elastic ground. The tunnel is assumed to deform radially into the shape described by a Fourier series due to tunneling. With the prescribed radial displacement and the calculated stress change, the radial subgrade modulus (k_r) can be obtained as follows:

$$k_r = \left.\frac{\sigma_r}{u_r}\right|_{r=r_0} \tag{2}$$

The assumptions made in this note are the same as those made by Wood (1975): (a) the plane strain condition is in a direction perpendicular to the cross section of the tunnel; (b)

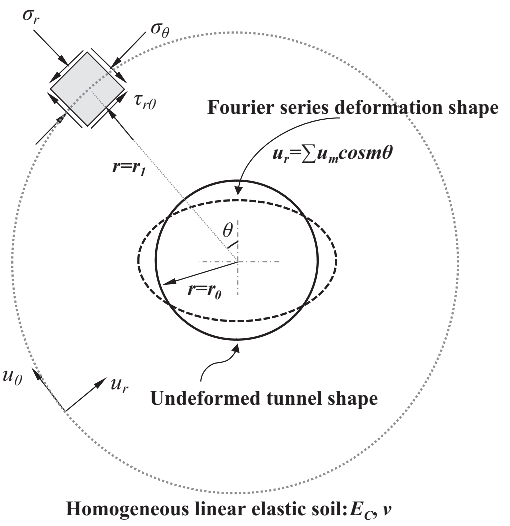

Fig. 1. Geometry of the problem.

Table 1
Parameter k_r recommended by standards in China and Japan.

Type of soil	Clayer or silty soils				Sandy soils			
	Very soft	Soft	Medium	Stiff	Very loose	Loose	Medium	Dense
k_r[a] (MPa/m)	3–15	15–30	30–150	> 150	3–15	15–30	30–100	> 100
$(k_r \times 2r_0)$[b] (MPa)	0–4	4–15	15–46	> 46	0–28			28–55

[a] China Standard (Liu and Hou, 1997).

[b] Japan Standard (RTRI, 1997): radial subgrade modulus (k_r) × tunnel diameter ($2r_0$).