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Prediction of plane strain undrained diffuse instability and strain localization with non-coaxial plasticity

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Abstract

In this paper, undrained diffuse instability and strain localization of frictional materials under plane strain conditions were studied. Based on a 3D non-associated Mohr–Coulomb hardening model, the theoretical criteria for diffuse instability and strain localization were proposed by the second-order work theorem and the vanishing of the determinant of the acoustic tensor. The criteria were used to predict the instability characteristics of soil specimen in isotropically and anisotropically consolidated plane strain tests. The studies showed that, when the soil specimen was loaded under strain-controlled loading mode, the soil becomes potentially unstable slightly before the shear stress reaches its peak value. The initiation of diffuse instability accompanies with the peak of shear stress, and strain softening occurs with further loading. Strain localization was predicted by the vanishing of the determinant of the acoustic tensor, and it is shown to occur after the diffuse instability. The non-coaxial plasticity flow rule was adopted to improve the prediction the onset of strain localization, while the inclusion of non-coaxial plasticity flow rule shows no influence on diffuse instability. Both diffuse instability and strain localization occur at the hardening stage of the soil and result in the reduction of the deviatoric shear stress.

Keywords: Diffuse instability; Strain localization; Non-coaxial plasticity; Undrained loading; Plane strain (IGC: D06)

1. Introduction

While failure, instability and bifurcation are typical problems in geomechanics, they are not synonymous (Chambon et al., 2004). The failure of soil occurs when the stress difference reaches a limiting value (Lade, 2002), and it is often characterized by Mohr–Coulomb plastic limit criterion. The stability problem is a widely discussed problem and has different definition: Bellman, 1953 pointed out that stability is a much overburdened word with an unstabilized definition. Lyapunov (1907) established the fundamental stability theory. From an engineering standpoint, a given system is stable if it is insensitive to small perturbations (Chambon et al., 2004), otherwise, instability behaviors, which correspond to the spontaneous change of the deformation mode in the next loading increment occurs. Bifurcation is a widely used theory in physics: it means the existence of more than one response path from a given state for the same loading path (Nicot and Darve, 2011). For non-associated materials, either strain localization or diffuse instability tend to occur before the Mohr–Coulomb plastic criterion is met. Strain localization (Alshibli and Sture, 2000; Desrues and Viggiani, 2004)

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Nomenclature

A	fitting parameter of hardening rule
A_d	potential function parameter
C^{np}_{ijkl}	non-coaxial compliance matrix
d_0	parameter of potential function
D	dilatancy function
D^{e}_{ijkl}	elastic tensor
D^{p}_{ijkl}	plastic tensor
D^{ep}_{ijkl}	elasto-plastic tensor
$(D^{ep}_{ijkl})_{sys}$	symmetric part of elasto-plastic tensor
E	elastic modulus
e_0	void ratio at initial state
e_{ij}	deviatoric strain
F	yield function
G	pressure dependent shear modulus
G_0	elastic shear modulus
$g(\theta_\sigma)$	shape function in deviatoric stress plane
H	hardening modulus
H_t	non-coaxial hardening modulus
J_2	second stress invariant
K	bulk elastic modulus
M	stress ratio
M_f	peak stress ratio
M_d	dilatancy stress ratio
p'	mean stress
p_{at}	atmospheric pressure
q	equivalent shear stress
Q	plastic potential
s	mean stress defined in 2D condition
s'_{ij}, S'_{ij}	effective deviatoric stress
t	shear stress defined in 2D condition
u	pore water pressure
β	parameter of the shape function
δ_{ij}	Kronecker delta
ε	mean strain in 2D condition
ε_{ij}	strain tensor
$\varepsilon_s, \dot{\varepsilon}_s$	equivalent strain and rate
$\dot{\varepsilon}^e_s$	equivalent elastic shear strain rate
ε^p_s	equivalent plastic strain
$\varepsilon_v, \dot{\varepsilon}_v$	equivalent volumetric strain and rate
$\dot{\varepsilon}^e_v$	equivalent elastic volumetric strain rate
γ	shear strain in 2D condition
ν	Poisson ratio
σ'_{ij}	total stress and effective stress tensors
σ'_{ij}	effective stress tensors
σ_1, σ_2, σ_3	principal stress
λ	plastic multiplier
θ	shear band angle
θ_σ	Lode angle

demonstrates deformation concentrated in narrow zones, diffuse instability (Ramos et al., 2012; Daouadji et al., 2011) usually happens in the case of homogeneous problem and there may be loss of homogeneity randomly distributed in space (but non localized pattern). The strain localization problem has been a hot topic and has been widely studied since 1970s, but diffuse instability was seldom explored until recent years (Khoa et al., 2006; Prunier et al., 2009; Laouafa et al., 2011).

Strain localization has been studied experimentally (Desrues and Viggiani, 2004), theoretically (Bardet, 1990) and numerically (Andrade and Borja, 2006; Lu et al., 2012), and has also been used in the progressive failure analysis of engineering problems (Zienkiewicz and Huang, 1995). The widely used theoretical framework for the prediction of strain localization is the localization criterion of Rudnicki and Rice (1975), Rice and Rudnicki (1980), which corresponds to the vanishing determinant value of the acoustic tensor. The predicted results from localization criterion significantly depend on the constitutive model. For the deficiency of conventional plasticity model in the prediction of strain localization, non-coaxial plastic flow rule is often necessary to improve the predicted results. Originated from the vertex-theory (Rudnicki and Rice, 1975), a non-coaxial Mohr–Coulomb elasto-plasticity model was formulated by Papamichos and Vardoulakis (1995) in the 2D condition to study the strain localization under the plane strain condition. In order to study the strain localization in plane strain tests as a 3D problem, the original 2D model has been extended to 3D case (Qian et al., 2008). This extended model was further modified by using an elaborated shape function to predict the strain localization on both isotropic and cross-anisotropic soils under true triaxial conditions (Huang et al., 2010; Lu et al., 2011). The influence of non-coaxial flow rule on the failure of soil has been also studied by the numerical modeling of soil element test and real engineering problems (Yang and Yu, 2010; Yang et al., 2011). These aforementioned researches on strain localization were based on the drained condition. However, previous experimental studies (Han and Vardoulakis, 1991; Finno et al., 1997; Chu and Wanatowski, 2009) and recent theoretical studies (Guo and Stolle, 2013; Noda et al., 2013a,2013b) have shown that strain localization also occurs under the undrained condition.

Diffuse instability is a phenomenon different from strain localization, and has been observed in drained tests with constant deviatoric shear (Chu et al., 2012) and undrained tests (Lade, 1994). Under the undrained condition, after the attainment of the peak deviatoric stress, the mechanical state of the soil element is potentially unstable, while in order to observe an instability (leading to a diffuse failure), a change in the loading mode is required (Nicot and Darve, 2011). As frequently observed in undrained triaxial tests (Lade and Pradel, 1990; Sladen et al., 1985) and plane strain tests (Wanatowski and Chu, 2012) on loose granular soil, the occurrence of the peak shear stress often corresponds to the onset of static liquefaction (Kramer, 1996; Vaid and Sivathayalan, 2000; Chu et al., 2003). The evaluation of the onset of static liquefaction was mostly based on the instability