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Natural frequencies for flexural and torsional vibrations of beams on Pasternak foundation

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Abstract

This paper studies the free vibrations of prismatic beams resting on Pasternak foundation. Special attention is given to the consideration of the bending–twist deformations of the beams. The governing differential equations of the motion are derived by imposing the dynamic equilibrium of a Timoshenko beam element. Differential equations are solved numerically using the combination of the Runge–Kutta and Regula–Falsi methods. The results of some cases are presented, and are analyzed to highlight the effects of the end constraint, rotatory and torsional inertias, aspect ratio, thickness ratio, beam stiffness, and foundation stiffness on the natural frequencies of the beams. The natural frequencies of the present model are validated by comparing to those from model tests.
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Keywords: Natural frequency; Vibration; Beam; Pasternak foundation; Torsion; Soil–structure interaction

1. Introduction

Beams resting on a ground surface that include footings and pipelines present very common soil–structure interaction problems encountered in geotechnical engineering. A good knowledge of the natural frequencies in dynamic problems is essential in the design of such structures, especially in the case of those subjected to dynamic loads generated by earthquakes, blasting waves and other sources (Morfidis, 2010; Allani and Holeyman, 2013; Ghazavi et al., 2013; Sapountzakis and Kampitsis, 2013a, 2013b). Therefore, a free vibration analysis is an important part in the total investigation of the system.

In a vibration analysis of the soil-supported beams, the soil medium is idealized by elastic foundations (see Fig. 1). The most common foundation model is the Winkler foundation (one-parameter model), shown in Fig. 1(a). In this model, the soil is represented by unconnected closely spaced linear elastic springs, which is simple but cannot reproduce the continuity characteristics of soils. To overcome this limitation, two parameter models have been developed (Filonenko-Borodich, 1940; Pasternak, 1954; Vlasov and Leontiev, 1960; De Rosa, 1995). A comprehensive overview on this topic is given by Dutta and Roy (2002). Kerr (1964) showed that the Pasternak foundation can be a possible mathematical model for the generalized foundation, which is also stated by other researchers (Guler, 2004; Calim and Akkurt, 2011; Maheshwari and Khatri, 2012). In the Pasternak model, a shear layer of incompressible vertical elements that resist only transverse shear is attached to the end of Winkler springs, as shown in Fig. 1(b). Thus, the two parameters of the foundation reflect the stiffness of the springs and the shear interaction between the springs.

Numerical studies have been carried out on the free vibration of beams resting on elastic foundations. Particularly

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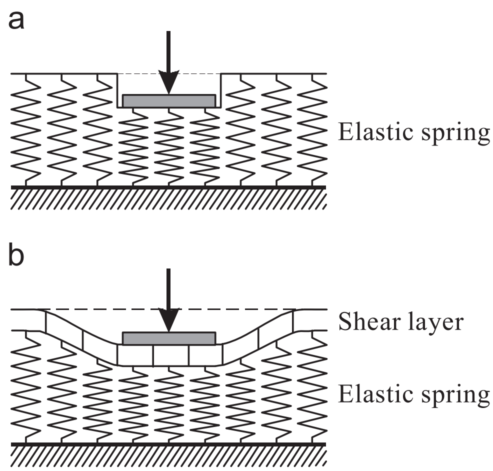

Fig. 1. Displacement of (a) Winkler and (b) Pasternak foundation models (modified from Selvadurai (1979)).

for beams supported by the Pasternak foundation, Wang and Stephens (1978) presented the general solution of natural frequencies for finite uniform beams with different end constraints and investigated the impact of Pasternak foundation on natural frequencies. Wang and Brannen (1982) examined the effect of open angle of vibrating curved beams on the Pasternak foundation. Yokoyama (1987) proposed a finite element method for analyzing the free vibration of shear and Timoshenko beams on Pasternak foundation. Eisenberger (1994) also applied the finite element method to find the exact vibration frequencies of cantilever beams on Pasternak foundation. Naidu and Rao (1995) highlighted the influence of initial stress on the vibration behavior of uniform beams on Pasternak foundation. El-Mously (1999) derived explicit formulae for the fundamental frequencies for the vibration of finite beams on finite Pasternak foundation using the virtue of Rayleigh's Principle. Matsunaga (1999) employed the one-dimensional higher order theory to compute the natural frequencies of beam–columns on Pasternak foundation. Chen et al. (2004) studied a mixed method that combines the state space method and the differential quadrature method to the free vibration of Euler–Bernoulli beams on Pasternak foundation. Ying et al. (2008) reported the exact solutions of flexural vibration of functionally graded beams on Pasternak foundation by two-dimensional elasticity theory. Zhu and Leung (2009) developed the hierarchical finite element for a nonlinear free vibration analyses of non-uniform Timoshenko beams on Pasternak foundation. The differential transform and dynamic stiffness matrix methods for a free vibration analysis of Pasternak foundation-supported beams were used by Balkaya et al. (2009) and Calio and Greco (2012), respectively. Li et al. (2012) dealt with transverse vibration of the shear beams containing rotatory inertia on Pasternak foundation. Most of the existing literature is devoted to capturing the flexural vibration along the beam. However, one may recognize that at the point of contact between beam and foundation, there is not only flexural deformation but also torsional deformation under the action of vibration. Few studies have been reported on the flexural–torsional vibration characteristics of beam–foundation system where twist against the cross-section of beams is taken into account, although there have been studies which considered the bending and twisting of beams (Rao and Mirza, 1988; Vo and Lee, 2009; de Borbon et al., 2011).

In this study, the flexural–torsional free vibrations of finite uniform beams resting on finite Pasternak foundation are described. The governing equations of the motion are deduced by considering the dynamic equilibrium of a Timoshenko beam element. A computer program coded in FORTRAN capable of calculating the natural frequencies of the system was developed. The results of several cases are presented and the influences of the end constraint, the rotatory and torsional inertias, the aspect ratio, thickness ratio, beam stiffness and the foundation stiffness on the natural frequencies are discussed. The natural frequencies from physical model tests are used to validate the present calculations.

2. Analytical model

Fig. 2 shows a Timoshenko beam resting on Pasternak foundation in a Cartesian coordinate system. As shown in Fig. 2(a), the beam is assumed to be straight and uniform, having a rectangular cross-section with width B and height H, which is small relative to its length L. The x-axis is oriented in the central axis of the beam, and the y and z-axes are in height and width directions, respectively. Various end constraints such as free, hinged and clamped ends can be taken as the end constraint of the beam. Fig. 2(b) shows a typical example of the vibrational mode shape for the beam. Under the action of free vibration, the beam deflects in the x–y plane, resulting in producing deflection v and rotation dv/dx, which are positive in the y direction and in the clockwise direction. Simultaneously, the beam also twists in the y–z plane, resulting in an angle of twist ϕ, which is positive in the clockwise direction. According

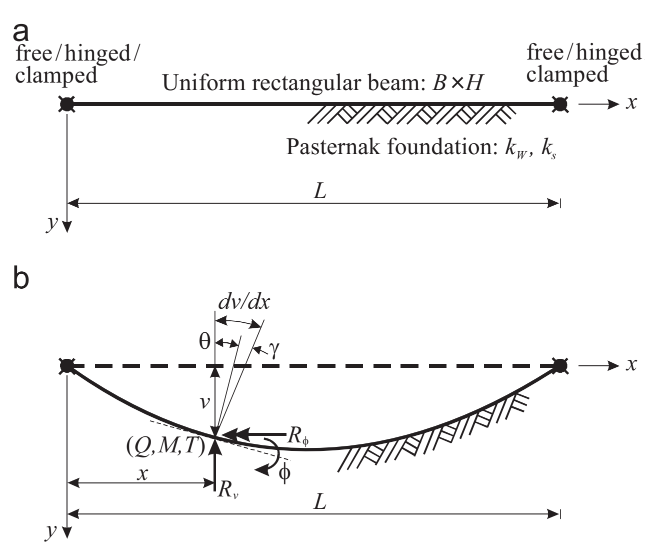

Fig. 2. Beam on Pasternak foundation in (a) undeformed and (b) typical mode shape.

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