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Realization of uniform deformation of soil specimen under undrained plane strain condition based on soil–water coupled finite deformation analysis considering inertia forces

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Abstract

Based on a soil–water coupled finite deformation analysis, theoretical considerations and numerical calculations were carried out under the undrained plane strain condition in order to reproduce a uniform deformation field. Rather than the "quasi-static" equation of motion, which does not include inertia forces, a dynamic equation of motion which includes inertia forces was used. At first, a theoretical consideration was carried out to realize uniform deformation for a saturated soil that satisfied the element-wise undrained/constant-volume condition. This presents an "infinitely slow loading" case without ignoring the inertia term based on the $\boldsymbol{u}$–p formulation. In other words, it can be seen that under general slow loading that is not infinitely slow, a gradient in the pore water pressure will always be produced, resulting in the migration of pore water and loss/collapse of uniformity. This first conclusion is useful for verifying numerical analysis code made in the finite deformation regime. Next, the uniform deformation of a plane strain rectangular soil specimen was measured under constant cell pressure and undrained boundary conditions using a dynamic soil–water coupled analysis in which the SYS Cam-clay model was employed as the elasto-plastic constitutive model for the soil skeleton. In addition, the effects of the loading rates as well as loading applications, with/without inertia forces, on the loss of uniformity in deformation were shown to have a significant influence on the inertia term even though the loss itself was extremely small.

Keywords: Inertia force; Loading application effect; Loading rate effect; Soil–water coupled effect; Uniform deformation; IGC: D06/E08

1. Introduction

Bifurcation and strain localization analyses of soil specimens are generally performed based on uniform deformation field under plane strain conditions, and when these numerical analyses are carried out, a quasi-static analysis that ignores the inertia term is often performed (for example, Hill and Hutchinson, 1975; Yatomi et al., 1989a, 1989b; Wan et al., 1990; Hashiguchi and Tsutsumi, 2003; Ikeda et al., 2003; Kimoto et al., 2004; Lu et al., 2011). In Asaoka and Noda (1995), the authors carried out compression tests on plane strain specimens provided with an initial geometric imperfection under a constant cell pressure and undrained boundary conditions using a quasi-static soil–water coupled analysis, in which the original Cam-clay model was used as the soil skeleton constitutive equation (Asaoka et al., 1994).

On the other hand, the authors expanded the quasi-static soil–water coupled finite deformation analysis that had been

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carried out and developed a code capable of taking the inertia term into consideration, without distinction between dynamic and quasi-static analysis (Asaoka and Noda, 2007; Noda et al., 2008). The main objective of this paper is to discuss the theoretical and numerical analyses for realization of uniform deformation based on finite deformation theory when problems that are conventionally treated as the "quasi-static" assumption are analyzed again as dynamic problems with the inertia term taken into consideration by integrating the equation of motion.

In this study, first, a uniform deformation field in saturated soil under undrained conditions was theoretically examined using a soil–water coupled analysis that considered the inertia term based on the $\boldsymbol{u}$–p formulation. It was found that even though the deformation was uniformly reproduced by applying initial values, there was still a gradient in the distribution of pore water pressure in the case where the loading rate was not "infinitely slow" (the meaning of this is explained below). At the same time, applying the initial values was useful for verifying the performance of the analysis code (Asaoka and Noda, 2007; Noda et al., 2008).

In the next main subject matter of this paper, the following two types of numerical analyses were carried out on plane strain rectangular specimens with a constant cell pressure and undrained boundaries. For this plane strain condition, the in-plane minor principal stress T_3 is kept constant and acts as the cell pressure on the four in-plane surfaces, while in the in-plane the major principal stress T_1 increases. Accordingly, the intermediate principal stress T_2, which is directed out-of plane, can be obtained by $T_2 = (T_1 + T_3)/2$ if initially isotropic stress state ($T_1 = T_2 = T_3$) is assumed, where T_1, T_2 and T_3 are explained in Eq. (4). Also for the constitutive model used in this analysis, which is called the SYS Cam-clay model (Asaoka et al., 2000, 2002), the effect of the intermediate principal stress T_2 is not taken into consideration. For ease of understanding the problems, the soil was a saturated and fully remolded clay and the gravity force was not considered in the analysis.

1. It is demonstrated that when compression deformation was applied at a vertical constant velocity under undrained conditions and a constant cell pressure to a perfectly rectangular specimen without material or geometrical imperfections, the analysis code (Asaoka and Noda, 2007; Noda et al., 2008) is capable of reproducing the uniformity of deformation. It should be noted that since it is practically impossible to satisfy such requirements for the laboratory sample preparation, this can be regarded as an ideal assumption in these theoretical and numerical calculations. In this case, in order to realize a uniform deformation, the vertical velocity of the top end was exerted proportionally to the height in the vertical direction from the bottom end and distributed inside the specimen; the distributed acceleration and pore water pressure were applied correspondingly to satisfy a uniform deformation field under undrained conditions as initial values. Consequently, only when an infinite slow loading rate (which can be defined as when the product of the permeability coefficient and the loading velocity is sufficiently small," details of which are formulated in Section 2) was satisfied, the uniform deformation within the rectangular specimen, like that in the conventional quasi-static analysis, was able to be reproduced. Moreover, the maintenance of uniformity also verified that even when the loading rate varied greatly, the coefficient of permeability is virtually zero.
2. Next, the initial condition in case 1, where the initial velocities, accelerations and pore water pressures were designated to reproduce the uniform deformation, was changed so that it was closer to the practical conditions. For simplicity, the calculation was carried out with initial zero values for velocity, acceleration and pore water pressure. As a result, the effect of the loading rate played a significantly more important role in the deformation patterns and in the deviator stress accompanying compression wave, all of which could not be observed in a quasi-static analysis.

2. Investigation into uniform deformation of saturated soil in a soil–water coupled field regarding inertia term

In this section, the general case in which a specimen is influenced by the inertia term is considered for an undrained compression test on a plane strain condition to investigate the possible situation for the uniform deformation in a soil–water coupled analysis. Consider a vertical extension/compression test with constant strain rate on a perfectly rectangular specimen under undrained conditions between rigid and frictionless pedestals at both the top and bottom ends. The Cartesian coordinate system is applied into the inertia system, with the reference coordinate in the horizontal and vertical directions, X_1 and X_2, respectively, the current coordinate in the horizontal and vertical directions, x_1 and x_2, respectively, and the uniform deformation field of the soil skeleton represented by the following equation:

$$x_1 = \frac{X_1}{\delta t + 1}, \quad x_2 = (\delta t + 1)X_2 \tag{1}$$

where $t \geq 0$ represents time, δ is a loading rate parameter indicating the rate of extension per unit time and per unit length in the vertical direction, $\delta > 0$ indicates extension deformation, and $\delta < 0$ indicates compression deformation. In this uniform deformation field, t is not simply a parameter for obtaining the velocity and acceleration but also indicates the actual time. Also, the domain over which time is defined is $0 \leq t < -1/\delta$ when $\delta < 0$ and is $0 \leq t \leq \infty$ when $\delta > 0$, and $x_1 x_2 = X_1 X_2$ shows that the specimen deforms element-wise in an undrained (constant-volume or no volumetric change) manner.

When the x_1 and x_2 components of coordinate are derived from the above equation, the following equations are obtained for velocity and acceleration:

$$\dot{x}_1 = -\frac{\delta}{(\delta t + 1)^2} X_1, \quad \dot{x}_2 = \delta X_2 \tag{2}$$

$$\ddot{x}_1 = \frac{2\delta^2}{(\delta t + 1)^3} X_1, \quad \ddot{x}_2 = 0 \tag{3}$$

where "$\cdot$" and "$\cdot\cdot$" above x_1 and x_2 indicate the first and second material time derivatives viewed from the soil skeleton,