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A multidirectional *p–y* model for lateral sand–pile interactions

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Abstract

The lateral loads applied to pile foundations, as induced by winds or earthquakes, are usually multidirectional. Experimental studies have indicated that the lateral resistance of the pile under multidirectional paths is generally lower than that under a unidirectional path and the degree of reduction depends on the characteristics of the loading paths. On the other hand, most currently used *p–y* models can take the soil–pile interaction under unidirectional lateral loading into account, but it cannot be applied directly to analyze the response of piles under multidirectional lateral loading. A multidirectional *p–y* model is proposed in this study, which is formulated within the framework of the bounding-surface elastoplastic theory and consists of two loading mechanisms: the parallel loading and the orthogonal loading. The model has five parameters, which are readily available or calibrated. To demonstrate its ability to model soil–pile interactions under both unidirectional and multidirectional lateral loadings, the proposed model is incorporated into a finite-element program to analyze laterally loaded piles. The responses of piles with different embedment lengths subject to various loading paths are investigated. The non-coaxial relationship between the force increment and the displacement increment vectors at the pile head under the multidirectional loading, and the impact of the multidirectional loading on the lateral resistance are well captured in the analyses.

Keywords: *p–y* Model; Multidirectional; Sand–pile interaction; Elastoplastic; Nonlinear

1. Introduction

Pile foundations, a type of deep foundation, have been widely used to support high-rise buildings, bridges and offshore structures. These foundations must be designed for lateral loads because the structures they support are subjected to such loads as the result of wind, waves and earthquakes. In the past several decades, a large number of in-situ and physical model tests (e.g., McVay et al., 1998; Dyson and Randolph, 2001; Kim et al., 2004; Lee et al., 2011; Tamura et al., 2012) have been performed to investigate the response of piles to lateral loads. Meanwhile, extensive analytical and numerical approaches (e.g., Ashour et al., 1998; Zhang et al., 1999; Hsiung, 2003) for investigating the behavior of laterally loaded piles have also been developed.

The load-transfer approach, often referred to as the "*p–y*" method, is one of the most popular approaches to determine the behavior of laterally loaded piles. In this approach, the pile is modeled as an elastic member while the soil is idealized as a discrete set of independent non-linear springs that describe the relationship between the local lateral soil–pile resistance and the lateral relative displacement. A number of *p–y* relationships have been proposed for sands (e.g., Reese et al., 1974; Wesselink et al., 1988), silts (e.g., Reese and Van Impe, 2001) and

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Nomenclature

p	soil resistance per area
y	lateral displacement of the pile
$\boldsymbol{p}$	vector of soil resistance per area
$\boldsymbol{y}$	vector of lateral displacement of the pile
$d\boldsymbol{y}_e$	elastic component of the displacement increment
$d\boldsymbol{y}_p$	plastic component of the displacement increment
k_e	elastic resistance coefficient
$d\boldsymbol{p}$	resistance increment
$d\boldsymbol{p}'$	part of resistance increment in parallel with the current resistance $\boldsymbol{p}$
$d\boldsymbol{p}''$	part of resistance increment orthogonal to the current resistance $\boldsymbol{p}$
$\boldsymbol{n}$	unit vector in parallel with $\boldsymbol{p}$
$d\boldsymbol{y}_p'$	plastic displacement increment induced by $d\boldsymbol{p}'$
k_{p1}	plastic resistance coefficient associated with the parallel loading
$\boldsymbol{m}$	direction of the plastic displacement increment induced by $d\boldsymbol{p}''$
p_u	ultimate soil lateral resistance
$d\boldsymbol{y}_p''$	plastic displacement increment induced by $d\boldsymbol{p}''$
k_{p2}	plastic resistance coefficient associated with the orthogonal loading
dl_1	loading index associated with the parallel loading
dl_2	loading index associated with the orthogonal loading
p_m	maximum soil resistance in the history
$\boldsymbol{\alpha}$	relocatable projection center on the bounding surface
ρ, $\overline{\rho}$	Euclidian distances
z	depth below soil surface
b	pile diameter
η_h	subgrade modulus
h_1	scaling factor for the modulus k_{p1}
σ_p	passive earth pressure of soil
c_p	model constant associated with p_u
κ	model constant associated with evolution of $\boldsymbol{m}$
y_r	referenced displacement ($y_r = p_u/k_{ini}$)
h_2	scaling factor for the modulus k_{p2}
E	Young's modulus of the pile
I	moment of inertia of area of the pile
dy_1, dy_2	pile displacement increments along the X_1 direction and the X_2 direction respectively
d_{p1}, dp_2	soil resistance increments along the X_1 direction and the X_2 direction respectively
l	element length
dy_j^i, $d\phi_j^i$	lateral displacement and rotation at the ith node ($i=1, 2, 3$) along the X_j direction ($j=1, 2$) respectively
dF_j^i, dM_j^i	external lateral force and moment at the ith node ($i=1, 2, 3$) along the X_j direction ($j=1, 2$) respectively
ks_{jk}^i	components of the stiffness matrix to take into account the multidirectional soil–pile interaction at the ith node
k_{jk}	components of the stiffness matrix of p–y model

clays (e.g., Matlock, 1970; Reese and Welch, 1975). However, most of these models were developed based on the observed behavior of piles subject to monotonic lateral loading. These models are generally semi-empirical and can only take into account the soil–pile interaction under unidirectional lateral loading.

In reality, the lateral loads and motions applied to pile foundations are often multidirectional, and in some cases the amplitudes of the two horizontal orthogonal components are comparable (Su and Li, 2008). Mayoral et al. (2005) conducted a series of model pile tests in clay under various multidirectional displacement paths. The results showed that the shape of the p–y curves is strongly affected by the loading path. Su (2011, 2012a) performed a number of laboratory-scale model tests on a pile–sand system under both unidirectional and multidirectional loading paths. The results indicate that the lateral resistance of the pile under the multidirectional path is generally lower than that under the unidirectional path. The degree of reduction depends on the characteristics of the loading paths.

As Su (2005) noted, the principle of superposition does not hold when the interaction between the pile and the surrounding soil is nonlinear. Therefore, most currently used p–y models cannot be directly applied to analyze the responses of pile foundations under multidirectional lateral loadings. This study proposes a multidirectional p–y model for laterally loaded piles in sand, formulated within the framework of the bounding-surface elastoplastic theory. The model is incorporated into the finite-element program Code for Analyzing Multidirectionally Loaded Piles (CAMLP) that was developed to analyze the response of piles under lateral loadings (Su, 2012b). This ability of this model to capture the key features of the soil–pile interactions under both unidirectional and multidirectional lateral loadings is demonstrated by comparing numerical simulation results to experimental observations.

2. Formulation of the multidirectional p–y model

2.1. Framework

In the model, $\boldsymbol{y}$ represents the lateral displacements of the pile at a given depth and $\boldsymbol{p}$ represents the soil resistance per area at the same depth. For a pile under multidirectional lateral loading, both $\boldsymbol{p}$ and $\boldsymbol{y}$ are vectors in a plane.