NeuroImage

www.elsevier.com/locate/ynimg
NeuroImage 39 (2008) 728–741

Variational Bayesian inversion of the equivalent current dipole model in EEG/MEG

Stefan J. Kiebel,[a,*] Jean Daunizeau,[a] Christophe Phillips,[b] and Karl J. Friston[a]

[a]*The Wellcome Trust Centre for Neuroimaging, Institute of Neurology, UCL, 12 Queen Square, London, WC1N 3AR, UK*
[b]*Centre de Recherches du Cyclotron, Université de Liège, Liege, Belgium*

Received 1 May 2007; revised 7 August 2007; accepted 3 September 2007
Available online 14 September 2007

In magneto- and electroencephalography (M/EEG), spatial modelling of sensor data is necessary to make inferences about underlying brain activity. Most source reconstruction techniques belong to one of two approaches: point source models, which explain the data with a small number of equivalent current dipoles and distributed source or imaging models, which use thousands of dipoles. Much methodological research has been devoted to developing sophisticated Bayesian source imaging inversion schemes, while dipoles have received less such attention. Dipole models have their advantages; they are often appropriate summaries of evoked responses or helpful first approximations. Here, we propose a variational Bayesian algorithm that enables the fast Bayesian inversion of dipole models. The approach allows for specification of priors on all the model parameters. The posterior distributions can be used to form Bayesian confidence intervals for interesting parameters, like dipole locations. Furthermore, competing models (*e.g.*, models with different numbers of dipoles) can be compared using their evidence or marginal likelihood. Using synthetic data, we found the scheme provides accurate dipole localizations. We illustrate the advantage of our Bayesian scheme, using a multi-subject EEG auditory study, where we compare competing models for the generation of the N100 component.

Keywords: EEG; MEG; Equivalent current dipole; Variational Bayes

Introduction

The analysis of evoked responses using magneto- and electroencephalography (M/EEG) can proceed in several ways. If one is interested in inferring the locations of M/EEG generators within brain space, one has to solve the inverse spatial problem (Baillet et al., 2001). There are two main approaches to estimating sources from observed sensor data. The first assumes that sensor data can be explained by a small set of equivalent current dipoles. The inversion of this model amounts to a nonlinear optimization problem, because the forward model is nonlinear in dipole location (Mosher et al., 1992). Recently, the source reconstruction problem has been addressed by placing many dipoles in brain space, and using constraints on the solution to make it unique; for example (Baillet and Garnero, 1997; Mattout et al., 2006; Phillips et al., 2005). This approach is attractive, because it produces images of brain activity comparable to other imaging modalities and it eschews subjective constraints on the inversion. For imaging solutions, most constraints can be motivated by anatomical and physiological arguments, *e.g.*, smoothness constraints and approximate location priors, based on regional activity in functional magnetic resonance imaging (fMRI). Traditional few-dipole solutions, however, are usually regarded as depending too much on user-specified modelling decisions; like the number of dipoles and their initial locations. Mathematically, it can be argued that the inversion of dipole models is a harder problem than inversion of distributed models, because the inverse problem of distributed source imaging is basically linear. These reasons might explain why much methodological research has been devoted to developing sophisticated Bayesian source imaging inversion schemes, while dipole models have received less such attention.

However, models with a few dipoles are useful, because they represent a direct mapping from scalp topography to a small set of parameters. Dipole solutions usually lend themselves to simple interpretations and provide an informative way to explain the observed data. Furthermore, it is easy to report the sufficient statistics of dipole parameters, over subjects. Operationally, summarising distributed activity with a small number of sources simplifies analyses of connectivity among those sources (*e.g.*, dynamic casual modelling of evoked or induced responses (Kiebel et al., 2006)). Critically, in a Bayesian context, different models can be compared using their evidence or marginal likelihood. This model comparison is superior to classical goodness-of-fit measures, because it takes into account the complexity of the models (*e.g.*, the number of dipoles) and, implicitly, uncertainty about the model parameters. For this reason, classical schemes have adopted

* Corresponding author. Fax: +44 20 7813 1420.
E-mail address: skiebel@fil.ion.ucl.ac.uk (S.J. Kiebel).
Available online on ScienceDirect (www.sciencedirect.com).

1053-8119/$ - see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.neuroimage.2007.09.005

other measures for model comparison (*e.g.*, the Akaike Information Criterion (AIC); see also Supek and Aine (1993) for an example using classical model comparison). For most models, the AIC and its cousin, the Bayesian Information Criterion (BIC) are a rough approximation to the model evidence (Beal, 2003; Penny et al., 2004), and are less accurate than the negative free energy. In this paper, we provide some examples of the usefulness of model comparison, with dipole models.

When the model comprises only one or two dipoles, the best solution can usually be found without using any constraints and a Bayesian framework appears to be superfluous. For three or more dipoles, model inversion is more difficult because many local minima of the high-dimensional objective function exist. In this situation, it is practically infeasible to visit all local minima and select the best solution. Rather, one can introduce constraints that preclude certain un-physiological solutions, and guide the inversion towards favoured solutions. Such constraints are implemented naturally using Bayesian techniques, but they invite criticism that using informative priors imposes a pre-selected solution. This criticism can be countered by observing that Bayesian model comparison allows one to assess several solutions objectively and assert that there is strong evidence in favour of a particular solution (Penny et al., 2004). Usually, in M/EEG, candidate models already exist, based on cognitive theories and preceding studies. These predictions motivate the use of informed priors, and the subsequent comparison of competing models. Therefore, Bayesian model comparison is a useful way to decide which theory explains the observed data best and informative priors are central to this strategy. Even inconclusive model comparison (*i.e.*, all models explain the data equally well) tells us the data do not provide enough evidence in favour of one theory over the other. These procedures and inferences could not proceed in a classical (*i.e.*, non-Bayesian) framework.

In short, fast Bayesian inversion for dipole models seems to be a useful addition to the toolbox for M/EEG analysis. In the present paper, we propose a variational Bayes (VB) inversion scheme for a single time point. Only a few Bayesian inversion schemes for (spatial or spatiotemporal) dipole models have been described in the literature (Auranen et al., 2007; Jun et al., 2005, 2006; Schmidt et al., 1999). These approaches are based on Monte Carlo–Markov chain techniques, which use time-consuming sampling procedures to compute the posterior distributions. Variational Bayes provides a fast and efficient approximation to the necessary integrals and has been applied successfully to source imaging in M/EEG (Daunizeau et al., 2007; Sato et al., 2004) and other problems in functional imaging (Flandin and Penny, 2007; Penny et al., 2003; Woolrich and Behrens, 2006).

There are other approximate optimization schemes that we could have used for implementing a Bayesian approach to dipole models. Among them are 'iterative conditional modes' (ICM) or conditional expectation–maximization algorithms. However, it is known that these techniques are not invariant under re-parameterizations while VB is. Furthermore, ICM does not per se compute the model evidence, which is easy to do with VB.

In the following, we first describe the equivalent current dipole model and derive the VB algorithm. In the second section, we use the VB and conventional scheme on synthetic and real data. We provide some examples of using informed priors and compare the two schemes. In the discussion, we address advantages, disadvantages and potential extensions of the approach.

Theory

Equivalent current dipole model

It is generally assumed that the bulk of remotely detected M/EEG signal is generated by synchronous depolarization of pyramidal populations, where the current flows between synapses proximate and distal to the cell bodies. The relationship between scalp data y and primary current density is linear and instantaneous so that

$$y = G(s)w \tag{1}$$

where $G(s)$ is the $(N_c \times 3N_s)$ lead-field matrix. N_c is the number of channels or sensors and N_s is the number of sources. The $(3N_s \times 1)$ vector location s forms the input argument for the nonlinear lead-field function, $G(s)$, whose output is multiplied by the $(3N_s \times 1)$ moment vector w to form the observed data.[1] The lead-field accounts for passive propagation of the electromagnetic field from the sources to the sensors (Mosher et al., 1999). Note that although the relationship between the data and primary current density is linear, it is non-linear in the dipole locations.

For EEG, a popular head model is based on four concentric spheres, each with homogeneous and isotropic conductivity. The four spheres approximate the brain, skull, cerebrospinal fluid (CSF) and scalp. The parameters of the model are the radii and conductivities for each layer. Here, we use radii of; 71, 72, 79 and 85 mm, with conductivities 0.33, 1.0, 0.0042 and 0.33 S/m, respectively. For MEG, one can use a single sphere as a good approximation. The potential or magnetic field at the sensors requires an evaluation of an infinite series, which can be approximated using fast algorithms (Mosher et al., 1999; Zhang, 1995). For the ECD forward model, we used a Matlab (MathWorks) routine that is freely available as part of the FieldTrip package (http://www2.ru.nl/fcdonders/fieldtrip/, see also Oostenveld, 2003) under the GNU general public license.

The observation model

We transform Eq. (1) into an observation model by adding an error term.

$$y = G(s)w + \varepsilon \tag{2}$$

We assume an independent and identically distributed (i.i.d.) normal error, which is parameterised by a precision parameter γ_y. This specifies a likelihood model for the data given the model parameters. The probabilistic generative model is completed by the specification of priors: The normally distributed parameter vectors, w and s, have gamma-distributed prior precisions γ_w and γ_s, which are scale parameters for prior covariance matrices Σ_{w_0} and Σ_{s_0} of the location and moment vectors. These do not need to be diagonal and can encode user-specified prior constraints (see below for illustrative examples). Fig. 1 shows the graphical model for this equivalent current dipole forward model, which will guide us in the subsequent derivation of update rules. We assume that the location and moment parameters are *a priori* independent of each other; that is, they are drawn independently of each other to generate the data. This precludes any prior correlation between location and moment, but such correlations are not used generally in ECD solutions.

[1] The moment vector can also be expressed as two angles and amplitude.