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Probabilistic analysis of shield-driven tunnel in multiple strata considering stratigraphic uncertainty

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ABSTRACT

Subsurface formations with multiple soil/rock strata are a common geological condition for shield-driven tunnel (i.e., tunnel constructed using shield-driven machines) construction. The excavation face, under such conditions, often encounters a frequently changing stratigraphic configuration that consists of various lithological units. Furthermore, due to a lack of direct and continuous observations of the subsurface region, it is difficult to predict the stratigraphic profile along the entire excavation path with a high degree of certainty. Such a widely changing and uncertain excavation environment may lead to wide variation in the state of stress and deformation of the tunnel structure along the longitudinal direction. This poses a challenge for design engineers in obtaining accurate performance evaluations or reasonable design outcomes for tunnel construction in subsurface ground with multiple strata. This paper aims to address this challenge by presenting a stochastic geological modeling framework for uncertainty quantification of stratigraphic profiles using sparsely located observation information from geotechnical site investigations. In the proposed modeling framework, the underground soil stratigraphic profile is regarded as a Markov random field with specific energy functions, which is able to describe the inherent anisotropic and non-stationary spatial correlation of lithological units in the subsurface stratigraphic structure. By incorporating the developed stochastic geological modeling framework with a finite element simulation of the tunnel excavation, a probabilistic analysis approach is established to evaluate the effects of stratigraphic uncertainty on the structural performance of a shield-driven tunnel.

1. Introduction

Shield-driven tunnel (i.e., tunnel constructed using shield-driven machines) construction typically involves excavation in subsurface regions with multiple lithological formations, as reported from engineering practice in Brazil [1], China [2], Korea [3] and Singapore [4]. Under such complex geological conditions, it is extremely difficult to obtain accurate subsurface geological information in advance, due to limitations of geotechnical investigation techniques and project budgets. Without accurate geological information along the tunnel alignment, conventional design based on analysis of subjectively selected critical cross sections may not always be conservative, even when a large factor of safety is adopted [5]. Thus, it seems that employing probabilistic analysis approaches that are able to incorporate geological uncertainties in tunnel construction is a logical step toward a more sensible analysis and design outcome.

For design and construction of geotechnical structures in multiple lithological strata, geological uncertainties originate from two main sources: interpretation of stratigraphic configuration and determination of material properties of each lithological formation. In the past, a considerable amount of effort [6–12] has been devoted to developing methods for determining the geomaterial properties and their spatial correlations in the probabilistic/reliability-based analysis and design for geotechnical structures. Such methods have been applied to assess risk for foundation systems [13,14], slope stabilization [15,16], and tunnels [17,18]. However, there has been relatively little effort spent in developing methods for quantifying stratigraphic uncertainty (i.e., uncertainties related to inference of the configurations of subsurface soil/rock profiles). A limited number of techniques utilizing geostatistical methods or variogram-based interpolation methods [19–23] can provide optimal but deterministic estimates of the contact boundaries

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between lithological formations. Nevertheless, these methods cannot provide the measure of probability/credibility of the interpreted stratigraphic configuration; thus, they are unable to provide a quantitative estimate of the stratigraphic uncertainty. Stochastic modeling frameworks, such as Markov chain model [24] and multiple-point geostatistics [25–28], are able to generate stratigraphic samples to form the corresponding probability distribution. However, these methods have evoked strong assumptions, such as stationary transition probability matrices or predefined data templates. Therefore, further development of methodologies for modeling stratigraphic configurations in a probabilistic manner and quantifying stratigraphic uncertainty are still needed.

The requirement of stratigraphic uncertainty quantification is further heightened in analysis and design of geotechnical projects related to long linear structures such as underground tunnels. With a length of thousands of meters, the surrounding stratigraphic configurations encountered by the tunnel excavation face can vary greatly from one location to another. Such variable and uncertain stratigraphic configurations can exert significant effects on a tunnel's structural performance (e.g. surface settlement, bending moment and convergence), as highlighted by numerous investigations including analytical methods, model experiments and numerical simulations [2,29,30]. Thus, there remains a critical need for developing a stratigraphic uncertainty quantification framework and a compatible probabilistic analysis approach that can be used for analyzing such long, linear geotechnical structures.

In this paper, we present a novel stochastic geological modeling framework for quantifying stratigraphic uncertainty for a large-scale geological body, conditional on available observation information. In the proposed modeling framework, a Markov random field (MRF) with a specific energy functions is employed to capture the anisotropic and non-stationary spatial correlation of lithological units in stratigraphic structures. By combining the presented modeling framework with finite element analysis (FEA), we establish a probabilistic analysis approach for an underground tunnel with consideration of stratigraphic uncertainty along the tunnel alignment.

This paper is organized in the following manner. In Section 2, we introduce the mathematical background of the MRFs and the proposed spatial correlation model in terms of potential functions for modeling the subsurface stratigraphic profile. In Section 3, a detailed description is provided to illustrate the stratigraphic uncertainty quantification process for a shield-driven tunnel example, based on the stochastic geological modeling framework. In Section 4, a series of stratigraphic configurations, as a probabilistic description for the real stratigraphic profile, are further input into an FEA program to calculate stress and deformation behavior of the studied shield-driven tunnel, which reveals the effects of stratigraphic uncertainty on the structural behaviors of the tunnel. Specific conclusions regarding the possible application of the proposed probabilistic analysis approach for an underground tunnel design, considering stratigraphic uncertainty, are given in Section 5.

2. Stochastic geological modeling framework

The subsurface stratigraphic profile can be regarded as a spatially correlated system of a configuration of several lithological units. In this study, an MRF prior is employed as a statistical model to describe the subsurface stratigraphic profile for its two favored features. First, the posterior distribution of an MRF is usually feasible and easily to obtained through efficient Markov chain Monte Carlo (MCMC) sampling methods. Second, the MRF prior can be specified by defining a certain potential function; therefore, it is convenient to implement the desired characteristics into an MRF to obtain the appropriate posterior outcome. With these attractive features, MRF priors have been successfully applied to the field of geoscience for handling problems, such as geological mapping [31] and land-cover classification [32]. In this section, we will introduce the mathematical structure of the MRF, as well as the proposed spatial correlation model in terms of potential functions for the stochastic modeling of subsurface stratigraphic profiles.

2.1. Neighborhood system, Markov random field and Gibbs sampler

For encoding a complete distribution over a two-dimensional space, an MRF is constructed on a graphic-based representation of the geological body using a certain discretization scheme. Herein, the domain of the stratigraphic profile is discretized into a finite set of elements $\boldsymbol{E}$, using a uniform square lattice. In such a meshed plot, the adjacent elements that share at least one node are regarded as neighbors to each other, and $\boldsymbol{N}_s$ denotes a local neighborhood system consisting of all the neighbors of a given element s. Next, the set of elements $\boldsymbol{E}$ and its neighborhood system $\boldsymbol{N} = \{\boldsymbol{N_s} | \boldsymbol{s} \in \boldsymbol{E}\}$ form a graph $\boldsymbol{G} = (\boldsymbol{E}, \boldsymbol{N})$ on which the MRF is built. In the MRF, we assign each element a lithological label from a finite set $\boldsymbol{L}$ of labels (representing all lithological units in the modeled domain) to form a random subsurface stratigraphic configuration $\boldsymbol{\omega} = \{\omega_s | s \in \boldsymbol{E}, \omega_s \in \boldsymbol{L}\}$. We use ω_s and $\boldsymbol{\omega_A}$ to represent the labels given by the configuration $\boldsymbol{\omega}$ to the element s and a subset of elements $\boldsymbol{A} \in \boldsymbol{E}$, respectively. Then, we denote a configuration space that contains all possible subsurface stratigraphic configurations as $\boldsymbol{\Omega} = \{\boldsymbol{\omega} = \{\omega_s\} | s \in \boldsymbol{E}, \omega_{\boldsymbol{s}} \in \boldsymbol{L}\}$. A prior joint probability $P(\boldsymbol{\omega})$ defined on $\boldsymbol{\Omega}$ is said to define an MRF if its local characteristics, referred to as the conditional probabilities $P(\omega_s | \boldsymbol{\omega}_{\boldsymbol{E}-\{\boldsymbol{s}\}})$, depend only on the labels of the neighboring elements, with the expression as follows:

$$P(\omega_s | \boldsymbol{\omega}_{\boldsymbol{E}-\{s\}}) = P(\omega_s | \boldsymbol{\omega}_{\boldsymbol{N}_s}) \tag{1}$$

The form of the conditional probabilities in Eq. (1) is the direct and clear mathematical description of the Markovianity of an MRF. However, it is unrealistic to formulate an MRF based on such a definition, since there is no obvious method available for deducing the joint probability from the associated conditional probabilities [33]. An alternate way of formulating the MRF is to represent it in terms of an equivalent Gibbs distribution according to the Hammersley–Clifford theorem [34]. Detailed proofs of such equivalence can be found in Besag [33] and Li [35]. In this way, the joint probability $P(\boldsymbol{\omega})$ of an MRF can be expressed in terms of a Gibbs measure $\pi(\boldsymbol{\omega})$:

$$P(\boldsymbol{\omega}) = \pi(\boldsymbol{\omega}) = \frac{1}{Z}\exp(-U(\boldsymbol{\omega})/T) \tag{2}$$

where $U(\boldsymbol{\omega})$ is the energy of configuration $\boldsymbol{\omega}$, which depends upon its label assignment. In Eq. (2), Z is a normalizing constant called the partition function and has the form:

$$Z = \sum_{\boldsymbol{\omega} \in \boldsymbol{\Omega}} \exp(-U(\boldsymbol{\omega})/T) \tag{3}$$

while T stands for "temperature" in the simulated annealing algorithm. The common form of T and relevant discussions can be found in Geman and Geman [34].

Although Eq. (3) provides the analytical form of the partition function Z, it is computationally intractable since the number of terms in this sum is typically too large. However, as Z is a constant, the *maximum a posteriori* (MAP) estimate $\tilde{\boldsymbol{\omega}}$ with the largest density of the joint probability $P(\boldsymbol{\omega})$, i.e.,

$$\tilde{\boldsymbol{\omega}} = \arg\max_{\boldsymbol{\omega} \in \boldsymbol{\Omega}} P(\boldsymbol{\omega}) \tag{4}$$

can be obtained through stochastic simulation on the configuration space $\boldsymbol{\Omega}$ using Gibbs sampling. As a specific MCMC sampling