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Structural analysis with alternative uncertainty models: From data to safety measures

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ABSTRACT

When adequate empirical data on uncertain variables is lacking, non-probabilistic approaches to quantify uncertainties become appropriate. This study discusses such situations in the context of structural safety assessment. The problem of developing convex function and fuzzy set models for uncertain variables based on limited data and subsequent application in structural safety assessment is considered. Strategies to develop convex set models for limited data based on super-ellipsoids with minimum volume and Nataf's transformation based method are proposed. These models are shown to be fairly general (for instance, approximations to interval based models emerge as special cases). Furthermore, the proposed convex functions are mapped to a unit multi-dimensional sphere. This enables the evaluation of a unified measure of safety, defined as the shortest distance from the origin to the limit surface in the transformed standard space, akin to the notion used in defining the Hasofer–Lind reliability index. Also discussed are issues related to safety assessment when mixed uncertainty modeling approach is used. Illustrative examples include safety assessment of an inelastic frame with uncertain properties.

1. Introduction

Probabilistic methods offer a powerful framework for quantitative treatment of uncertainty in structural engineering problems. Thus, problems of uncertainty propagation, local/global response sensitivity analyses, time variant/invariant reliability estimation, reliability based design, code calibration, risk analysis, system identification, and health monitoring can be formulated within the framework of these methods. In spite of these capabilities, it is important to note that the credibility of probabilistic models crucially hinges upon the availability of adequate amount of empirical data to construct probabilistic models for the uncertain quantities. If one is confronted with paucity of empirical data, alternative modeling strategies, classified as non-probabilistic methods, which involve interval analysis, convex function modeling, and/or fuzzy set theory, become more appropriate. One could also envisage situations in which both probabilistic and one or more non-probabilistic tools can be combined in treating uncertainty within a single problem.

Some of the earliest studies which dealt with the application of interval analysis to structural mechanics problems are due to Koyluoglu et al. [1], Dimarogonas [2], and Rao and Berke [3]. The study by Koyluoglu et al., considers development of finite element models for linear static skeletal structures with interval properties and interval valued loads. Conservative bounds on the solutions are obtained by using triangle inequality and linear programming tools. Dimarogonas [2] considered the interval eigenvalue problem arising in modal analysis of uncertain systems and discussed the response of uncertain rotors systems. Rao and Berke [3] have considered a few approaches to tackle linear equations of the form $[A]X = B$ with $[A]$ and B being interval valued. These include methods based on rules of interval arithmetic, a combinatorial approach, and a truncation based scheme aimed at limiting the growth of intervals in calculations involving large systems and large uncertainties. Some of the other contributions here include studies on beam systems [4], comparison of interval finite element based solutions and stochastic finite element analysis for static systems [5], and for dynamical systems [6], analysis of transfer functions of dynamical systems [7], efforts on sharpening bounds obtained using interval arithmetic [8], developing guaranteed bounds on response [9], studies on actively controlled vibrating systems [10], analysis using component mode synthesis [11], applications to problems of structural optimization [12], development of perturbation based approaches [13], problems of system identification [14], and dynamic response analysis using response spectrum method [15].

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Extensive discussion on convex models is available in the monographs by Ben-Haim and Elishakoff [16], and Ben-Haim [17,18] and also in the review papers by Elishakoff [19,20], and Ben-Haim [21]. The range of problems considered here includes studies on loads induced on a vehicle as it moves on an uneven substrate, bounding response of dynamical systems excited by inputs which are bounded by total mean square energy, optimal placement of sensors, bounds on dynamic response of systems with imperfect geometric characteristics modeled as convex functions, stability of such geometrically imperfect systems [22], optimization of truss structures under uncertain loads modeled as ellipsoid convex sets [23], and a comparison on probabilistic and convex function modeling in the context of problems of dynamic buckling of bars [24]. The possibility of the analysis, based on convex models, providing a means to formulate a criterion for controlling manufacturing procedures to achieve bounded variability on quantities, such as, geometric dimensions, has also been pointed out in these studies.

Yao and Furuta [25] introduced the idea of employing tools of fuzzy sets to deal with uncertainties in civil engineering problems. Dynamic analysis of structural systems with excitations modeled as random processes and system parameters as fuzzy sets have been studied by Chiang et al. [26]. Modeling of crossing rate and reliability in such setting has been discussed. In a review article, Elishakoff [20] has outlined conceptual issues when finite element modeling is used for uncertain structures with probabilistic, fuzzy and interval models. In a series of papers, Rao and his group [27–29], employed fuzzy set theory to several problems including studies on vibration analysis of skeletal structures, nonlinear dynamics of rotors due to imbalance, and problems of structural damage detection. Studies on linear and nonlinear system of equations with fuzzy parameters and their relevance to problems of finite element analysis of uncertain structures have been reported by Skalna et al. [30]. The use of fuzzy sets in treating epistemic uncertainties in finite element models has been investigated by Hanss and Turrin [31]. Similar applications related to modeling of epistemic uncertainties have been discussed by Reuter and Schirwitz [32]. The monograph by Moller and Beer [33] contains extensive account of treatment of fuzzy random variables in the context of structural safety analysis. The uncertainty model here enables treatment of both fuzzy uncertainty and aleatoric randomness within a single mathematical framework.

If more than one framework for uncertainty modeling (*viz.*, probabilistic, interval, convex or fuzzy approaches) is used in a single problem, the mathematical operations involving the propagation of these uncertainties need to take into account different rules of arithmetic and evaluation of functions of uncertain variables. Joslyn and Ferson [34] examined some of the related issues. Chakraborty and Sam [35] have considered issues related to representing non-probabilistic uncertain variables as equivalent probabilistic variables and have subsequently used probabilistic methods for safety analysis. The equivalence of converting from one form of uncertainty model to the other is based on the application of entropy based principles and scaling of membership functions. Anoop et al. [36] consider conversion of probabilistic model into equivalent fuzzy variables. Discussion on the application of mixed uncertainties in the context of inverse reliability analysis can be found in the work of Balu and Rao [37]. When non-probabilistic uncertainty models are employed, the definition of structural safety measures has been a subject of research on a few publications. Thus, the studies by Ben-Haim [38,17,21] develop the notion of robust reliability. This involves the definition of a performance function (as in probabilistic approaches) and introduction of a variable α which signifies the magnitude of uncertainty in load and (or) system parameters. A measure of safety is given by the largest value of α for which failure becomes possible. The work of Cremona and Gao [39] fashions procedures, for safety measure evaluation in a non-probabilistic framework, which are akin to those used in calculation of reliability indices in probabilistic framework. In an insightful paper, Langley [40] has proposed a unified treatment of safety assessment with alternative uncertainty models and has shown that, no matter which framework for uncertainty modeling is used, the safety measure can be obtained by solving a constrained optimization problem. The author has discussed problems as relevant to forward and inverse reliability analyses.

The problem of arriving at possibilistic models for uncertain variables starting from limited data has not received wide attention in the existing literature. The studies by Zhu et al. [41] and Wang et al. [42] focus on finding optimal hyper-rectangle or ellipsoids with minimum volume which enclose the available data. In a given situation, the criterion to adopt to assess which method of uncertainty modeling best represents the available data has been discussed by Wang et al. [42]. While hyper-rectangle and ellipsoidal geometries offer simple alternatives to fit the data, they are not necessarily the only feasible choices. We investigate in the present study alternative convex function models for limited data which use the theory of super-ellipsoids (see, for example, Bardinet [43]) and method of Nataf's transformation that is widely used in structural reliability modeling [44]. It is pointed out that the use of super-ellipsoids has the potential to unify modeling approaches using intervals, ellipsoidal convex functions, and more general class of convex models. The recent studies by Elishakoff and Bekel [45], and Elishakoff and Elettro [46] investigate the use of super-ellipsoids with constant shape parameters in the context of convex modeling of empirical data. The study by Langley [40] demonstrates how alternative measures of safety, based on possibilistic models for uncertainties, can be quantified as solutions of an associated constrained nonlinear optimization problem that is akin to problem encountered in the definition of the Hasofer–Lind reliability index. We explore these ideas further and introduce the notion of a standard space in which the possibilistic model for uncertainty is mapped to a sphere of unit radius. The shortest distance from the surface of this sphere and the transformed performance function is deemed as the measure of safety and the determination of this quantity is shown to be mathematically equivalent to the problem of determination of the Hasofer Lind reliability index. The question of characterizing safety measures when mixed uncertainty models which combine probabilistic and convex function approaches to model uncertainties is discussed. Alternative measures for quantifying safety in such contexts are outlined. In order that the safety assessment procedures can be applied to practical problems of interest, computer programs developed for the calculation of safety measures (on Matlab platform) are combined with finite element models for structural behavior developed on professional softwares such as Abaqus.

2. Development of convex and fuzzy set models from limited data

Consider a $N \times 1$ vector $X = (X_1 X_2 \dots X_N)^T$ which is deemed to represent uncertain variables in a given problem and let M realizations of X be available. This data set is denoted by a $N \times M$ matrix d with its elements denoted by d_{ij}; $i = 1, 2, \dots, N, j = 1, 2, \dots, M$. It is assumed that the number of data points M is not adequate enough to arrive at an accepted probabilistic model for the vector X. Consequently, it is aimed to explore if a convex function or a fuzzy set model for X can be developed based on the data d. We consider two alternative procedures for constructing the convex model: the first is based on modeling the volume enclosed by the data in N dimensional space by an optimal super-ellipsoid processing the