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Estimate of small first passage probabilities of nonlinear random vibration systems by using tail approximation of extreme distributions

Jun He *, Jinghai Gong

Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

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ABSTRACT

Efficient estimate of the small first passage probabilities of nonlinear structures under random excitations is of great importance in structural reliability analysis. Some methods, such as the subset simulation method, tail equivalent linearization method, asymptotic sampling method, Monte Carlo simulation method based on the extreme value theory, etc., have been developed for estimating the probabilities, however, the efficiency of the estimate is still a challenging task. In the present study, a new method is developed to overcome the challenge. The method approximates the tails of the univariate extreme distributions of the responses by using the shifted generalized lognormal distributions, in which the model parameters are estimated by an efficient method called the extrapolation method. Based on the approximate tail distributions and covariances of the extreme responses, the tails of the multivariate extreme distributions of the nonlinear response are determined by using the Nataf model. Finally, from the relationship between the first passage probabilities and the extreme value distributions, the small first passage probabilities of interest can be estimated. The efficiency of the developed method is illustrated by the small first passage probability analyses of a single-degree-of-freedom system subjected to a stationary Gaussian process and two connected electric substation equipment items subjected to earthquake base motion.

1. Introduction

The efficient estimate of the small probabilities of failure, i.e., small first passage probabilities, of nonlinear structural systems under random excitations is of great importance and also a challenging task. The methods developed for the problem, such as the Monte Carlo (MC) simulation method [1], equivalent linearization method [2], Fokker–Planck equation method [2], stochastic averaging method [3], moment closure method [2], path integration method [4], probability density evolution method [5], etc., cannot efficiently estimate the small probabilities of first-passage failure of nonlinear random systems due to the their respective limitations. Therefore, the subset simulation method [6], tail equivalent linearization method [7], asymptotic sampling method [8] and MC simulation methods based on the extreme value theory [9,10] have been developed recently. These methods have been successfully applied to a large variety of nonlinear structure systems under random excitations, however, they also have some limitations in practical applications. For example, the subset simulation method requires a new simulation run when a different and smaller reference time is considered, the tail equivalent linearization method needs to repeatedly determine the equivalent linear systems when the response of interest is at the nonstationary state or a different response is considered, the asymptotic sampling method may lead to the large discrepancies of the estimates of the small failure probabilities if the unsuitable support points are chosen, and the MC simulation methods based on the extreme value theory require that the reference times are sufficiently long such that the extreme values of the responses of interest can be assumed to follow the generalized extreme value distributions approximately. Thus, there is a need for more efficient methods to be developed for estimating the small probabilities of failure, i.e., first passage probabilities, of nonlinear structure systems under random excitations.

In fact, for nonlinear structures under random excitations, the first passage probabilities of the response processes are identical to the corresponding exceedance probabilities of the extreme values of the responses [2,10]. Therefore, the small probabilities of failure can be estimated if the tail distributions of the extreme responses can be determined. As mentioned above, the generalized extreme value distribution has been adopted to fit the extreme values of the nonlinear random responses with sufficiently long reference times [9,10]. However, in the cases of short reference

* Corresponding author.

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times, the assumption that the extreme value responses follow the generalized extreme value distribution is invalid and, consequently, the resulting small probabilities of failure may exhibit large numerical errors [11]. Hence, more suitable distributions should be chosen to fit the extreme response distributions, particularly in the tail regions, such that the small probabilities of failure can be estimated with enough high accuracy.

More recently, a new distribution called the shifted generalized lognormal distribution (SGLD) for fitting four moments has been developed [12]. The distribution has a rich flexibility in shape that nearly encompasses the entire skewness–kurtosis region permissible for unimodal densities and has been applied to fit several theoretical distributions and actual datasets with very favorable results, particularly in the tail regions. If we can show that the SGLD is suitable to fit the tail distributions of the extreme values of responses of nonlinear structures under random excitations and its parameters can also be efficiently estimated, then the challenge of estimating the small probabilities of failure of nonlinear structures under random excitations can be overcome to some degree.

In the present study, mainly based on the SGLD, we will develop an efficient method to estimate the tail distributions of the univariate and multivariate extremes of responses of nonlinear structures under random excitations, from which the small probabilities of failure of the nonlinear random vibration systems can be efficiently estimated. The efficiency of the developed method will be illustrated by two numerical examples, including a hysteretic oscillator subjected to a stationary Gaussian process and a mechanical system modeling two connected electric substation equipment items under the action of random earthquake load.

2. Brief description of the SGLD

The intention of the SGLD is to construct a unimodal distribution density under four moment constricts. The distribution is developed by combining the three-parameter lognormal distribution [13] and generalized Gaussian distribution (GGD) [14]. The three-parameter lognormal distribution is an asymmetrical distribution and it and its reverse counterpart encompass the entire range of skewness, while the GGD is a symmetrical distribution and encompasses the entire range of kurtosis. Therefore, through synthesizes the features of the two distributions, the SGLD has a high flexibility in shape, nearly encompassing the entire skewness–kurtosis region permissible for unimodal densities. To define the SGLD, one should always consider the absolute value of skewness of the involved variable. If the skewness is less than zero, then through mirroring the resulting probability density function (PDF) about the mean value of the variable one can obtain the intended PDF. Similarly, through mirroring the resulting cumulative probability function (CDF) about the mean value of the variable one can obtain the intended CDF when the involved skewness is less than zero.

The PDF and CDF of a SGLD variable X with the positive skewness are given by, respectively

$$f_X(x) = \frac{\alpha}{x-b}\exp\left(-\frac{1}{r\sigma^r}\left|\ln\frac{x-b}{\theta}\right|^r\right),\ x > b \tag{1}$$

$$F_X(x) = \frac{1}{2} + \frac{1}{2}\operatorname{sgn}\left(\frac{x-b}{\theta} - 1\right)\varrho\left(\frac{1}{r}, \frac{\left|\frac{\ln\frac{x-b}{\theta}}{\sigma}\right|^r}{r}\right),\ x > b \tag{2}$$

where b is the location parameter, θ is the scale parameter, $0 < \sigma$ and $0 < r$ are the shape parameters, the coefficient α is defined as $\alpha = 1/[2r^{1/r}\sigma\Gamma(1+1/r)]$, here $\Gamma(\cdot)$ denotes the gamma function, sgn $(\cdot)$ denotes the signum function, and ϱ is the lower incomplete gamma function ratio, i.e., $\varrho(s,z) = \int_0^z t^{s-1}e^{-t}dt/\Gamma(s)$.

The above PDF, i.e., Eq. (1), is unimodal and asymmetric for $\sigma > 0$. In the limiting case $\sigma \to 0$ the SGLD converges to the GGD. Other special/limiting cases of the SGLD model are: when $r = 2$ and $b = 0$, the SGLD reduces to the lognormal distribution; when $r = 2$ and $b \neq 0$, the SGLD reduces to the 3-parameter lognormal distribution; when $r = 1$ and $b = 0$, the SGLD reduces to the Log-Laplace distribution; whereas for $r \to \infty$ and $b = 0$ the SGLD converges to the Log-uniform distribution.

By making the substitution $p = F_X(x)$ in the left-side-hand of Eq. (2) and noting that $\operatorname{sgn}((x-b)/\theta - 1) = \operatorname{sgn}(p - 1/2)$, the inverse CDF of the SGLD is obtained as

$$x = F_X^{-1}(p) = \theta\exp\left\{\operatorname{sgn}\left(p-\frac{1}{2}\right)\sigma\left[r\varrho^{-1}\left(\frac{1}{r}, \frac{2p-1}{\operatorname{sgn}\left(p-\frac{1}{2}\right)}\right)\right]^{1/r}\right\} + b,\quad \text{for } p \neq \frac{1}{2} \tag{3}$$

and $x = b + \theta$ for $p = 1/2$. Where, ϱ^{-1} is the inverse of the lower incomplete gamma function ratio, defined such that $z = \varrho^{-1}(s,\omega)$ corresponds with $\omega = \varrho(s,z)$.

Although the SGLD is a 4-parameter distribution, the parameter estimation of a SGLD only involves two variables, i.e., the shape parameters σ and r, since for each fixed pair (σ, r) the location and scale parameters can be computed from $\theta = \sigma_X/\sigma_Y$ and $b = \mu_X - \theta\mu_Y$, where μ_X and σ_X are the mean value and standard deviation of X and μ_Y and σ_Y are the mean value and standard deviation of the reduced variable $Y = (X-b)/\theta$, which can be computed from the fixed σ and r by the following raw-moments formula [12]

$$E[Y^k] = \frac{1}{\Gamma(1/r)}\sum_{n=0}^{\infty}\frac{(k\sigma)^{2n}}{(2n)!}r^{2n/r}\Gamma\left(\frac{2n+1}{r}\right) \tag{4}$$

For the case that the involved skewness is less than zero, one first needs to consider the absolute value of the skewness and determine the model parameters b, θ, σ and r, from which one obtains the PDF and CDF, i.e., Eqs. (1) and (2). Then, through replacing x by $2\mu_X - x$ on the right-hand-side of Eq. (1) one obtains the intended PDF of X. To obtain the intended CDF of X, one can replace x by $2\mu_X - x$ on the right-hand-side of Eq. (2) and the resulting expression is subtracted from unity. The intended inverse CDF of X with the negative skewness can be then derived from the obtained CDF:

$$x = F_X^{-1}(p) = 2\mu_X - \theta\exp\left\{\operatorname{sgn}\left(\frac{1}{2}-p\right)\sigma\left[r\varrho^{-1}\left(\frac{1}{r}, \frac{1-2p}{\operatorname{sgn}\left(\frac{1}{2}-p\right)}\right)\right]^{1/r}\right\} - b,\quad \text{for } p \neq \frac{1}{2} \tag{5}$$

and $x = 2\mu_X - \theta - b$ for $p = 1/2$.

Note that the SGLD has a support $[b, +\infty)$ for the case that the involved skewness is larger than zero and a support $(-\infty, 2\mu_X - b]$ for the case that the involved skewness is less than zero.

3. Evaluation of the small first passage probabilities of a scalar response process

The reliability analysis of a nonlinear structure under random excitation often involves the evaluation of the small first passage probabilities of a scalar response process within the reference time T. Without loss of generality, in this study we consider the first passage probabilities of a component response process $|X_k(t)|$ of the