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An improved weighted average simulation approach for solving reliability-based analysis and design optimization problems

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ABSTRACT

The weighted average simulation technique is one of the newest techniques that showed promising capabilities in solving structural reliability problems. In this paper, a modification to the weighted average simulation method is proposed, which allows the computation of the reliability with a small number of performance function evaluations. In addition, an approach for conducting reliability-based design optimization problems by introducing an additional improvement to the weighted average simulation method is proposed. This improvement significantly reduces the number of performance function evaluations needed in the process without sacrificing the accuracy of the results. It was found that the proposed approach can obtain the same results as the original weighted average simulation method with only a fraction of the computational cost required.

1. Introduction

The topic of reliability analysis has attracted significant attention due its capability of handling uncertainty in a rational manner. Engineering design codes have long been calibrated with means of reliability analysis. Accordingly, designers implicitly consider the uncertainties in the design variables and parameters by using the code safety factors calibrated using reliability analysis. Some of the current research efforts in the field are geared toward explicitly integrating reliability analysis concepts within the design process. Such approaches have not yet become state of the practice, despite the advances achieved in the field.

The importance of considering uncertainties during the engineering design process drives the need for developing computationally efficient techniques that enable engineers to achieve optimal and reliable designs [1]. The solution of reliability-based design optimization (RBDO) problems is a complex process. It requires the application of computationally efficient reliability analysis methods. First Order Reliability Method (FORM) is one of the widely used reliability analysis methods developed thus far [2]. This method approximates the performance function by a hyperplane which is tangent to the failure surface at the most probable point, which is the point that has the highest likelihood among all points in the failure region. The accuracy of FORM becomes questionable in cases where the performance function has multiple minimum points, and when the performance function is not linear in the vicinity of the design point [3].

To improve the accuracy of FORM, Fiessler et al. [4] introduced the Second Order Reliability Method (SORM) which approximates the performance function by a quadratic hypersurface in the neighborhood of the most probable point. Even though SORM is the most known improvement of FORM, numerous other methods were proposed over the past few decades [5–12]. Each of these methods has its own limitations. For instance, the accuracy of SORM may decrease in cases that include when the performance function is highly nonlinear, when there is a large number of random variables in the problem, or when the probability of failure is very small [3,13–15].

Sampling methods, such as Monte Carlo Simulation (MCS) and importance sampling, can be used to evaluate reliability. However, sampling is impractical if the performance function evaluation is time consuming or if the probability of failure is very small [3,16,17]. Many practical engineering applications have performance functions that are computationally expensive and time consuming to evaluate. Since the performance function must be evaluated for all generated samples, the computational cost of sampling techniques renders their use to be prohibitive in many engineering applications.

Rashki et al. [18] proposed a simulation based approach for approximating the probability of failure and the most probable point of failure. In this paper, this method is referred to as the

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weighted average simulation method (WASM). The premise of the method is that uniformly distributed samples are first generated in a design space for all random variables and the probability density value is applied as the weight index at each sample. The probability of failure is then computed by dividing the sum of the weight indices of the samples located in the failed region by the sum of the weight indices of all samples [18]. This method is highly accurate in estimating small values of the probability of failure and it only needs a few samples to obtain a solution [18–21].

The goal of an engineering design is to achieve adequate safety with minimum cost as well as the need to fulfill given performance requirements [22]. With optimization, one can establish the most economical system while satisfying predetermined safety and performance constraints [23]. Typically, deterministic conventional optimization is used in practice. Optimum structures obtained using deterministic optimization procedures may not have high reliability [24,25]. If it is intended that reliability analysis becomes a key tool in structural optimization, treatment of uncertainty in the optimization process is necessary [26]. The RBDO is a very appropriate and advantageous approach for design under uncertainty, as it incorporates reliability measures within the optimization process.

The integration of simulation based reliability methods in RBDO solution procedures significantly increases the computational cost of solving the problem. This issue is exacerbated when the technique is applied to practical and large scale engineering problems, where the evaluation of the performance function is computationally intense.

The WASM in its original form can be utilized in an RBDO problem, where it is run every time the probability of failure needs to be calculated. However, such approach can also be time consuming when performance functions are computationally expensive. Instead, Rashki et al. [19] proposed an approach for using the WASM to solve RBDO problems requiring only one simulation run at the beginning of the optimization solution. They showed that the nature of weighting and sampling in the WASM provides the opportunity to compute new failure probabilities by using the results of the first simulation run. However, the application of this approach is limited to RBDO problems with a small number of design variables and to problems with design variables that are all treated as random variables [19]. Assuming deterministic design variables as random variables with small coefficient of variations could lead to a conservative solution by the proposed method; unless the total number of design variables becomes too large.

In this paper, a modification to the WASM approach is proposed which allows the computation of the reliability with a small number of performance function evaluations. It is shown that with this modification, the probability of failure rapidly converges to the final result and only a fraction of the generated samples will need to have their performance functions evaluated.

In addition, in this paper, an approach for conducting RBDO problems by introducing an additional improvement to the WASM is proposed. When reliability-based constraints are introduced in RBDO problems, the actual value of the probability of failure associated with the limit state included in this constraint is irrelevant in some optimization methods. What is only relevant is whether the design point is feasible or infeasible with respect to the constraints associated with this limit state. The proposed approach applies checkpoints within the improved WASM that terminate the simulation process as soon as one of these checkpoints determines the state of the design point, i.e., if it is feasible or infeasible. This improvement significantly reduces the number of performance function evaluations needed in the RBDO process without sacrificing the accuracy of the results. Furthermore, the approach may be used with problems having deterministic design variables or mixed deterministic and random variables, as opposed to Rashki et al.'s method [19], which is only applicable to problems with design variables that are all random. The firefly algorithm [27–31] is used for the optimization solution. It was found that the proposed approach can obtain the same results as the original WASM with only a fraction of the computational cost required.

2. The weighted average simulation method, WASM

The WASM is a simulation based approach for determining the probability of failure by using the concept of the weight index for the generated samples. Also, an index function is used to distinguish the samples located in the failed region from those in the safe region. The probability of failure is then determined as the sum of the weight indices for the samples located in the failed region divided by the sum of the weight indices for all samples [18].

The first step in the WASM is to determine the proper intervals for each random variable in the problem. One of the proposed methods to determine these intervals in Rashki et al. [18] is to use a MCS to generate samples from each random variable. The minimum and maximum values of the generated samples from a design variable are used as the lower and upper points, respectively, for the interval of that random variable. The required number of samples generated by MCS can be determined by initially assuming a probability of failure and calculating the number of samples required to determine this assumed probability of failure [18]. Rashki et al. [18] showed that WASM can provide the same results even if the sample generation interval is changed, provided that increasing the interval length requires a larger number of samples to cover the design space and achieve accurate results. Additional details and another method to determine the sample generation interval can be found in [18].

The second step is to generate samples in a random variable space for all random variables. The uniform distribution is used for the generation of the random samples in the determined intervals. Then, the weight index is determined for each sample following its generation. The product of the probability density functions (PDFs) of the variables is applied to calculate the weight index of the samples as follows [18]

$$W(i) = \prod_{j=1}^{s} f_j(i) \tag{1}$$

where $W(i)$ is the weight index of the ith sample, s, is the number of random variables, and f_j is the PDF of the jth variable. Eq. (1) is applicable only to statistically independent random variables. For the case where the random variables are statistically dependent, the joint probability density function can be used as the weight index of the ith sample [18].

An index function, $I(i)$, for the ith sample is then determined by evaluating the performance function, g_i, at sample i and the result is established a follows [18]

$$I(i) = \begin{cases} 1 & \text{if} \quad g_i < 0 \\ 0 & \text{if} \quad g_i \geqslant 0 \end{cases} \tag{2}$$

Finally, the probability of failure, P_f, is calculated as [18]

$$P_f = \frac{\sum_{i=1}^{N} I(i) \cdot W(i)}{\sum_{i=1}^{N} W(i)} \tag{3}$$

where N is the number of samples.