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Reliability analysis of structures by iterative improved response surface method

Somdatta Goswami, Shyamal Ghosh, Subrata Chakraborty *

Department of Civil Engineering, Indian Institute of Engineering Science and Technology Shibpur, Howrah, India

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ABSTRACT

The moving least-squares method (MLSM) is a more accurate approach compare to the least-squares method (LSM) based approach in approximating implicit response of structure. The advantage of MLSM over LSM is explored to reduce the number of iterations required to obtain the updated centre point of design of experiment (DOE) to construct the final response surface for efficient reliability analysis of structures. The initial response surface is constructed based on a simplified DOE with mean values of the random variables as the centre point and updated successively to obtain the improved response surface. The reliability of structure is evaluated using this final response surface. The basis of the efficiency of the proposed method hinges on the use of simplified DOE instead of computationally involved full factorial design to achieve desired accuracy. As MLSM is more accurate compare to LSM in evaluating response surface polynomial, the centre point obtained is expected to be more accurate during iterations. Thus, the number of iteration in the update procedure will reduce and the accuracy of computed reliability will also improve. The improved performance of the proposed approach with regard to efficiency and accuracy is elucidated with the help of three numerical examples.

1. Introduction

Uncertainty in the parameters characterising the mechanical behaviour of a system and loads acting on it calls for reliability analysis. The reliability assessments require computation of probability of failure. Several methods are available to estimate the probability of failures. Those can be classified into two groups: analytical and simulation based methods. In the first group, one can find the well-known second moment based First Order Reliability Method (FORM) and Second Order Reliability Method. In the second group, the methods based on Monte Carlo Simulation (MCS) can be found. All such methods are now established and well documented in numerous texts [1–5]. These reliability analysis methods involve repetitive evaluations of performance function and it can be carried out directly so long structural response is possible to obtain explicitly. When a closed-form expression of the performance function is available, the number of performance function calls does not play an important role. On the contrary, when a finite element (FE) model is involved to obtain the structural responses, each performance function evaluation may require enormous computational time, especially when complex nonlinear constitutive behaviours are involved [6]. Thus, assessing the reliability of a complex structure requires a transaction between the reliability algorithms and numerical methods used to model the mechanical behaviour of the system.

The second moment based algorithms require computation of gradients and hessians of performance function. For implicit performance function, finite difference methods are usually adopted for approximating the gradients of the performance functions. This requires a large number of numerical computations. Furthermore, the second moment methods cannot always provide desired accuracy, particularly when the levels of uncertainty in the parameters are relatively large. Whereas in direct MCS approach, repeated evaluation of performance function involves large number of executions of the FE model of a structure. Thus, development of approach requiring fairly low computational time becomes important; particularly for safety assessments of large complex systems. Various techniques requiring fewer samples such as the importance sampling, directional simulation, antithetic varieties etc. are proposed in the literatures [4,7,8]. Though the number of samples involved in such techniques is lower than that required by the former one, still it remains important especially when analysis of complex structures by FE model is involved [9]. Furthermore, the mechanical model and the reliability evaluation algorithm need to be merged together for the safety assessment

* Corresponding author.
E-mail address: schak@civil.iiests.ac.in (S. Chakraborty).

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of complex problems. For such merging, the structural FE models are often constructed by using commercial FE software [10]. Thus, during each simulation one needs to run the FE software which limits the popular applications of FE method for practical structural reliability analysis problem. Therefore, alternative techniques for efficient computation of response of complex structures by overcoming aforementioned drawbacks while retaining the accuracy is of paramount importance to structural engineering community. Response surface method (RSM) based metamodelling technique has emerged as an effective solution to such problems. The RSM represents a convenient way to achieve a balance between the number of execution of the FE model and the accuracy of computed reliability. The present study deals with efficient evaluation of reliability of structures in the framework of iterative RSM.

2. Response surface method and reliability of complex system

RSM is one of the most sparking developments in structural reliability analysis. It is highly suitable for the case where the closed form expression for the performance function is not known and need to be evaluated by numerical methods such as FE method. The use of RSM, originally proposed by Box and Wilson [11], is still subject of further researches [12–14]. The developments that have taken place in the field can be studied under three subheads: (i) the studies dealing with experimental design in the physical space or in the standard normal space for a better control of the distance between numerical experiments [15–21], (ii) the studies related to the forms of response surface functions i.e. use of polynomial response surface like simple linear and quadratic response surface [15–17], use of neural network [10] etc. and (iii) the studies on various methods of design of experiment (DOE) [15–18]. The application of RSM for structural reliability analysis was first proposed by Faravelli [22] and the subsequent works [15–18] set the tone for its application for reliability analysis of large and complex structural system. For constructing response surface various DOE are used e.g. saturated design (SD), factorial design, central composite design (CCD) etc. It is well known that the accuracy of evaluating a performance function and its gradient largely depends on the capability of a metamodel to capture the nonlinearity and local variation of response behaviours. Though, the CCD approximates far better compared to other designs, it requires enormous input information involving structural response analysis compared to other experimental design approaches. Thus, in order to construct an efficient response surface requiring less number of FE executions is much desirable for large complex structures characterised by too many input variables.

The position of the sample points, the type of polynomial response and its performance is the subject of investigation by several authors and is still under discussion. It is well known that in reliability evaluation of structure using RSM, the centre point of design to construct the response surface should be close to the most probable failure point (MPFP) so that the response surface obtained includes most of the failure region with sufficient accuracy. In fact, an improved RSM has been suggested based on this fact [15] and studied the effectiveness of estimating reliability of structures using simple SD based RSM. Rajashekhar and Ellingwood [18] further suggested improvement by repeated application of the approach to obtain a converged centre point to construct the final response surface. Allaix and Carbone [23] used an iterative strategy to determine a response surface that is capable to fit the limit state function in the neighbourhood of the design point. The locations of the sample points are decided according to the importance sensitivity of each random variable to evaluate the free parameters of the response surface. These studies applied the least-squares method (LSM) to evaluate the unknown coefficients of response surface polynomial. The level of accuracy possible to achieve and the number of iterations required to obtain the converged centre point of design depends on the type of problems, the nature of variation of the responses and their sensitivities with respect to uncertain parameters. The number of iterations required may be large in many cases; thereby reducing the efficiency of the RSM significantly. But considering the fact that the moving least-squares method (MLSM) is much more accurate in evaluating response surface polynomial [24], the centre point obtained by the MLSM is expected to be more accurate during iterations. It has been applied to update a linear response surface successively by adding one new experimental point to the previous set of experimental points [25]. However, starting with linear response surface and adding only one extra point during iteration may not be accurate enough. The present study tries to explore the advantage of the MLSM based RSM compare to LSM based RSM to reduce the number of iterations to obtain the converged centre point of the DOE to construct the final response surface so that reliability analysis can be performed on this final response surface more efficiently. The contribution aims in proposing an efficient strategy for reliability analysis of structures following the basic iterative algorithm of Rajashekhar and Ellingwood [18] by judicious application of the MLSM based RSM instead of conventionally adopted LSM based RSM. The basis of the efficiency hinges on the use of a simplified DOE instead of computationally involved full factorial design usually sought to achieve desired level of accuracy. The initial response surface is constructed based on the SD considering the mean values of the random variables as the centre point. Now this centre point is updated using Bucher and Bourgund [15] algorithm until convergence. However, the response surface polynomial coefficients are obtained by MLSM to obtain the converged centre point of the DOE to construct the final response surface. The reliability of the structure is evaluated based on this final response surface. The improved performance of the proposed approach with regard to efficiency and accuracy is elucidated with the help of three numerical examples.

3. LSM based improved response surface for reliability analysis

The RSM is a set of mathematical and statistical techniques designed to obtain a better understanding about the overall response by DOE and subsequent analysis of experimental data. The method primarily uncovers analytically complicated or implicit relationship between several inputs and desired output through empirical models (non-mechanistic) in which the response function is replaced by a simple function (often polynomial) that is fitted to data at a set of carefully selected points (referred as DOE). In a sense, RSM is a system identification procedure, in which a transfer function relating the input parameters (loading and structural system parameters) to the output parameters (response in terms of displacements, stress, etc.) is obtained in a suitable way. The basic procedure to obtain a response surface that will serve as a surrogate for the FE model requires calculating the predicted values of the response features at various sample points selected as per the experimental design.

If there are n response values y_i corresponding to n numbers of observed data, x_{ij} (denotes ith observation of the input variable x_j in a design), the relationship between the response and the input variables can be expressed as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{y} \tag{1}$$

In the above multiple non-linear regression model $\mathbf{X}$, $\mathbf{y}$, $\boldsymbol{\beta}$ and $\boldsymbol{\varepsilon}_\mathbf{y}$ are the design matrix containing the input data from the design, the response vector, unknown co-efficient vector and error vector,