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A new method for reliability assessment of structural dynamic systems with random parameters

Jun Xu *

Department of Structural Engineering, College of Civil Engineering, Hunan University, Changsha 410082, PR China
Hunan Provincial Key Lab on Damage Diagnosis for Engineering Structures, Hunan University, Changsha 410082, PR China

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ABSTRACT

The reliability of structural dynamic systems involving random parameters can be assessed based on the equivalent extreme value distribution of response. To derive such probability density function, a new method is proposed in the present paper, in which an iterative scheme is involved. In each iterative step, an estimated solution is easily obtained firstly by solving a linear system of equations and the more accurate result is then searched around the estimated solution with high efficiency. The derivation is based on the principle of maximum entropy, in which an un-constrained optimization problem with fractional moments is involved. Three different methods for numerical integration of fractional moments are compared, which indicates that the bivariate dimension reduction method ensures the accuracy and efficiency simultaneously. Two examples, of which one deals with a linear random structure subjected to seismic excitation, the other deals with a nonlinear structure with random parameters subjected to random ground motions, are illustrated to validate the proposed method. The investigations show that the proposed method is of satisfactory accuracy and applicable to the reliability assessment of practical structural dynamic systems.

1. Introduction

The determination of the reliability of a dynamically excited structure is a key issue in civil engineering because of the inherent uncertainties in dynamic loads and in modeling of the structure [1]. The definition of failure of a structural dynamic system is usually based on the first passage [2–4] over the prescribed threshold of a structural dynamic response. For the first passage problems, the reliability was usually evaluated based on the level-crossing theory through Rice's formula or by the diffusion theory through the Kolmogorov equation [5]. However, it is always difficult to obtain the joint probability density function (PDF) of the target response and its velocity needed in Rice's Formula [2] to obtain the mean-level crossing rate. Besides, the Possion's assumption [6] or the Vanmarcke's assumption [7] are always adopted to describe the properties of the level crossing events, which both comes more from intuition and empirical data than from strong theoretical basis [8]. On the other hand, though the diffusion theory method was applied to a nonlinear single-degree-of-freedom (SDOF) system [9], its application to multiple-degree-of-freedom (MDOF) systems still remains to be a challenging task, which needs prohibitive computational efforts and therefore actually unfeasible in practical engineering.

Further, the first passage reliability assessment can be transformed to be an equivalent extreme value event such that the reliability evaluation becomes an integration over a one-dimensional equivalent extreme value distribution (EEVD) [8]. It is important to note that the correlation information is inherent in the equivalent extreme value (EEV). The present paper is devoted to evaluate the EEVD of structural dynamic response for reliability analysis. Therefore, two keys issues in the reliability assessment are to: minimize the number needed for deterministic dynamic response analysis and find the most appropriate probability distribution for the EEV.

Recently, the maximum entropy (MaxEnt) principle with fractional (not integers) moments has been received great interest to characterize the probability density function of a random variable [10,11] or static reliability analysis of structures [12]. A finite number of fractional moments, which embody a large number of central moments, are able to characterize the probability distribution of a random variable [13]. In fact, the EEV of structural dynamic response is a random variable, the distribution of which could be evaluated with the aid of fractional moments. On the other hand, due to the complexity of a structural dynamic system,

* Address: Department of Structural Engineering, College of Civil Engineering, Hunan University, Changsha 410082, PR China. Tel.: +86 18874015667.
E-mail address: xujun86@hnu.edu.cn

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especially a system exhibiting strong nonlinearity, the response function may be very complicated and irregular [14] in practical situations. Therefore, the fractional moment evaluation of dynamic response, which needs to consider the computational efforts and the accuracy, is of paramount importance to capture the EEVD for reliability analysis. Though some approaches have been developed for static cases [12], they may not provide the same satisfactory results in dynamic scenarios.

The present paper is to develop a new method for reliability assessment of a general nonlinear structural dynamic system based on the EEV, the MaxEnt principle and the fractional moments, in which the tradeoff of accuracy and efficiency is of great concern. This paper is organized as follows. A brief review of the EEVD for reliability assessment of structural dynamic systems is given in Section 2. In Section 3, the proposed method to derive the EEVD is presented based on the MaxEnt principle with fractional moments as constrained conditions. Different numerical integration methods are compared in Section 4 to obtain the fractional moments involved in the proposed method with consideration of the balance between efficiency and accuracy. Numerical cases studies are carried out in Section 5 to verify the proposed method. In Section 6, the concluding remarks are included.

2. The equivalent extreme value distribution based reliability assessment

Without loss of generality, an MDOF nonlinear structural dynamic system with random parameters can be expressed as

$$\mathbf{M}(\boldsymbol{\Theta})\ddot{\mathbf{X}}+\mathbf{C}(\boldsymbol{\Theta})\dot{\mathbf{X}}+\mathbf{G}(\boldsymbol{\Theta},\mathbf{X})=\mathbf{F}(\boldsymbol{\Theta},t) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the $n\times n$ mass and damping matrices, $\mathbf{G}$ is $n\times 1$ linear/nonlinear restoring force vector, $\mathbf{F}$ is the $n\times 1$ external force vector, $\boldsymbol{\Theta}=\left(\Theta^1,\Theta^2,\ldots,\Theta^d\right)$ denotes the random vector represented by a total of d independent random variables involved in both structural parameters and external excitations because the dependent random variables can be transformed to be independent ones by some specific transformations, e.g. Rosenblatt transformation [15], the known probability density function (PDF) of $\boldsymbol{\Theta}$ gives $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})=\prod_{j=1}^{d}p_{\Theta_j}(\theta_j)$. The deterministic initial conditions of Eq. (1) is

$$\mathbf{X}(t_0)=\mathbf{x}_0,\quad \dot{\mathbf{X}}(t_0)=\dot{\mathbf{x}}_0 \tag{2}$$

For a well posed dynamic system, the solution to the system (1) exists, is unique and must be a function of $\boldsymbol{\Theta}$. It is convenient to assume the solution takes the form [16]

$$\mathbf{X}=\mathbf{H}(\boldsymbol{\Theta},t) \tag{3}$$

Likewise, the velocity of $\mathbf{X}$ gives

$$\dot{\mathbf{X}}=\mathbf{h}(\boldsymbol{\Theta},t) \tag{4}$$

where $\mathbf{h}=\partial\mathbf{H}/\partial t$.

For dynamic reliability analysis, $Z(t)$, which could be considered as the stress at key point, the inter-storey drift, etc. could be determined by the state vector via some physical laws,

$$Z(t)=\boldsymbol{\Psi}\left[\mathbf{X}(t),\dot{\mathbf{X}}(t)\right]=\mathbf{H_Z}(\boldsymbol{\Theta},t) \tag{5}$$

where $\boldsymbol{\Psi}$ is the operator that transforms the state vector to be the physical quantity of interest for reliability analysis.

In general, the first-passage dynamic reliability with a single failure mode is defined by [14]

$$R=\Pr\{Z(t)\in\Omega_S,\quad \forall t\in[0,T]\} \tag{6}$$

where Pr denotes the probability, Ω_S is the safe domain, $[0,T]$ is the time duration.

For example, if a doubled sided boundary condition is considered, Eq. (6) is equivalent to

$$R=\Pr\{|Z(t)|<z_B,\quad \forall t\in[0,T]\} \tag{7}$$

where z_B is the safe boundary.

If one wants to obtain the reliability directly in Eq. (7), the infinite-dimensional joint PDF of the stochastic process $Z(t)$ is of necessity where the correlation information among different time instants is required [16]. Unfortunately, the information of the correlation is always unavailable in most situations. Alternatively, another form of the reliability assessment of structural dynamic systems is introduced.

Theorem 1. *Suppose $X_1,X_2,\ldots,X_m$ are m random variables. Let $W_{\max}=\max_{1\leqslant j\leqslant m}(X_j)$, then it goes that [8]*

$$\Pr\left\{\bigcap_{j=1}^{m}(X_j<a)\right\}=\Pr\{W_{\max}<a\} \tag{8}$$

where $W_{\max}$ is called the "Equivalent Extreme Value" (EEV).

According to the situation similar to Theorem 1, if the EEV is defined as

$$W_{\max}=\max_{t\in[0,T]}(|Z(t)|) \tag{9}$$

then, the reliability in Eq. (7) is equivalent to

$$R=\Pr\{W_{\max}<z_B\}=\int_{-\infty}^{z_B}p_{W_{\max}}(w)dw \tag{10}$$

where $p_{W_{\max}}(w)$ is the equivalent extreme value distribution (EEVD).

Likewise, if there are multiple limit state functions involved, say

$$R=\Pr\left\{\bigcap_{j=1}^{m}\left(|Z_j(t)|<z_B\right),\quad \forall t\in\left[0,T_j\right]\right\} \tag{11}$$

where $[0,T_j]$ is the time duration corresponding to Z_j, the corresponding EEV of Eq. (11) goes to

$$W_{\max}=\max_{1\leqslant j\leqslant m}\left(\max_{t\in[0,T_j]}\left(|Z_j(t)|\right)\right) \tag{12}$$

Thereby, the reliability in Eq. (11) can be determined straightforwardly by Eq. (10).

It is worth pointing out that the total information of correlation is inherent in the EEV, which has been proved rigorously in Ref. [8]. It is seen that viewed from the EEVD, the problem of dynamic reliability evaluation is transformed to be a simple integration problem, which requires neither the joint PDF of the response and its velocity, nor the assumption on properties of the level-crossing events. Therefore, the task of reliability assessment of structural dynamic systems is to evaluate the EEVD of the response. To this end, the method that provides a powerful tool for estimating a general PDF is of great significance to be investigated.

3. Maximum entropy principle based evaluation of equivalent extreme value distribution using fractional moments

Recently, the maximum entropy (MaxEnt) principle has gained increasing attention for reconstructing density distributions, where the moments are required as the constrained conditions [17,18]. As is known from the probability theory that the PDF can be determined once all the integer central moments of a random variable are known if they are all finite [19]. However, practical difficulties with the MaxEnt method appear when a relatively large number of moments of stochastic dynamic response of general nonlinear structures, say order $\geqslant 6$, are required [20,21,12]. Therefore, the fractional moments, which embody the information