Structural Safety 59 (2016) 1–8

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Robust design with Variability Response Functions; an alternative approach

Odysseas Kokkinos *, Vissarion Papadopoulos

Institute of Structural Analysis and Seismic Research, National Technical University of Athens, 9 Iroon Polytechneiou, Zografou Campus, Athens 15780, Greece

ARTICLE INFO

Article history:
Received 17 June 2015
Received in revised form 26 October 2015
Accepted 26 October 2015
Available online 14 November 2015

Keywords:
Variability Response Functions
Stochastic finite element analysis
Robust Design Optimization
Stochastic dynamic systems
Multi objective Genetic Algorithm

ABSTRACT

In this work a different Robust Design Optimization (RDO) approach is proposed implementing the concept of Variability Response Function (*VRF*), which is a function that when combined with the power spectral density of the stochastic field that models the system's uncertainty, formulates an integral expression for the variance of the system's response. The basic idea is to exploit a very well-known property of the *VRF*, which is its independence of the stochastic system parameters, in order to obtain global optima that depend only on the deterministic parameters of the problem. This way, optimal structural designs are achieved which are globally insensitive to uncertainties, that is to say they are free of the spectral-distribution characteristics of the stochastic fields modeling the uncertainties. This is achieved by setting in addition to the total material cost, the maximum *VRF* value as an objective function. The advantages of using the proposed methodology over traditional Robust Design Optimization are illustrated through an application to a frame-type structure where it is demonstrated that the designs achieved through classical RDO for a given stochastic field description are not optimal for a variation on the spectral properties of the random field modeling the system uncertainty, while optimal designs obtained with the *VRF*-based RDO remain optimum for the worst case scenario stochastic fields.

1. Introduction

The concept of Robust Design Optimization (RDO) has been introduced in order to deal with intrinsic uncertainties in physical systems that drive the system performance to deviate from the deterministically expected performance into sub-optimal designs, thus neutralizing the effort of the optimization procedure itself. In RDO the analyst is taking into account the stochastic properties of the system variables/parameters and/or system constraints and effectively reaches a safer optimum design which should be less sensitive to random system parameter variations. Various methodologies have been proposed in recent years regarding RDO and its applications to various problems. In classical RDO formulation the goal of minimizing objective function(s) is achieved by considering the mean and/or the standard deviation of a response quantity and trying to establish the designs that minimize the aforementioned quantities considering deterministic or reliability constraints [1,2]. In Reliability-based Robust Design Optimization (RRDO) [3–5] usually care is taken to address the influence of probabilistic constraints as a limit on the probability of failure in the context of RDO of structures. Vulnerability-based Robust Design Optimization (VRDO) [6] is a special case of RRDO where intermediate limit states approaching the probabilistic constraints are also taken into account thus providing possibly crucial information regarding structural behavior and operational integrity.

All previously mentioned RDO formulations are to be carried out in a stochastic finite element method (SFEM) framework so as to efficiently estimate the required quantities associated with system variations. This consideration of system randomness however, for it to be reliable, requires a precise knowledge of probabilistic characteristics (marginal *pdf's* and correlation structures) of the respective random fields modeling system parameters acquired only through corresponding experimental surveys or otherwise careful assumption/selection of various statistical properties describing the system variables/parameters uncertainty. Furthermore it increases substantially the analysis computational cost as any candidate design requires full stochastic analysis for the estimation of various statistical quantities. In the frequent case that such conditions are not met, similar analyses are implemented based on sensitivity analyses with respect to the aforementioned parameters resulting in a significant further increase of the overall computational cost.

In the present paper an alternative RDO procedure is proposed utilizing Variability Response Functions (*VRF*) concept [7–15] in an

* Corresponding author.
E-mail addresses: okokki@mail.ntua.gr (O. Kokkinos), vpapado@central.ntua.gr (V. Papadopoulos).

http://dx.doi.org/10.1016/j.strusafe.2015.10.001
0167-4730/

effort to provide an answer in aforementioned known issues while optimizing a frame structure involving stochastic field material properties with respect to its total weight and robustness of its displacement response. It is reminded here that system response variance, as originally proposed in [7] and then extended and further developed in [8–14], can be expressed in the following integral form expression:

$$Var(\mathbf{u}) = \int_{-\infty}^{\infty} \mathbf{VRF}(\kappa, \sigma_{ff})\ S_{ff}(\kappa)d\kappa \tag{1}$$

In the above expression σ_{ff} is the uncertain system variable standard deviation, $S_{ff}(\kappa)$ is the stochastic field spectral density and κ the spatial frequency (rad/m). **VRF**'s product and integration with the spectral density function $S_{ff}(\kappa)$ of the stochastic field that models the uncertain system variable(s) amounts to system response variance vector $Var(\mathbf{u})$. In the above expression **VRF**, which is a vector comprised of a *VRF* for each degree of freedom of the FE system, is assumed to be deterministic, an assumption proven rigorously only for statically determinate beam-type structures. For a number of other applications this assumption has been demonstrated numerically while further evidence has been provided with the introduction of the so called Generalized *VRF* (*GVRF*) which is a *VRF* calculated from a family of spectral density functions and various *pdfs*. What is really beneficial under this assumption is the ability to establish spectral- and *pdf*-free upper bounds in a straightforward manner described in the following equation as it has been explained in [12]:

$$Var(\mathbf{u}) \leqslant \mathbf{VRF}(\kappa^{\max}, \sigma_{ff})\ \sigma_{ff}^2 \tag{2}$$

where $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$ is the maximum value of the *VRF* attained at some wave number $\kappa^{\max}$. Therefore, setting maximum *VRF* value as an objective function accounting for system response robustness, in addition to the total weight, the system is ensured to exhibit, for a given weight class, the lowest possible variance response under conditions imposed by the worst possible stochastic field. The worst possible stochastic field for a particular design candidate is determined by means of Eq. (2) i.e. it is a stochastic field with a monochromatic *SDF* concentrated at $\kappa^{\max}$ [12]. The optimum design candidate for this particular weight class is the one that minimizes the respective $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$ value. Repeating this process for all possible weight classes one a two dimensional Pareto front is created for two objective functions: the weight and the system variance response accruing from Eq. (2).

In classical RDO formulation, optimization is performed for an a priori selected stochastic field. In real life applications however correlation structure of the uncertain system parameter is rarely known thus rendering such an optimization procedure redundant. Consequently the designer is obliged to conduct multiple such optimization procedures to shield the designed system from all possibilities. By using the proposed methodology this problem is overcome because each design candidate is evaluated based on its performance under the worst case scenario determined for the specific design. Effectively the designer is ensured that the system will have the best possible performance at the worst possible conditions.

The advantages of using the proposed methodology over traditional Robust Design Optimization are illustrated through an application to a frame-type structure where it is demonstrated that the designs achieved through classical RDO for a given stochastic field description are not optimal if a variation on the spectral properties of the random field modeling the system uncertainty occurs. On the other hand optimal designs obtained with the VRF-based RDO remain optimum for the worst case scenario stochastic fields. In order to demonstrate this, a bi-objective function is formulated taking into account uncertainties in the material properties modeled as random fields. Deterministic constraints of maximum stress and displacement response are applied. A Pareto front is initially constructed through a classical RDO formulation and multi-objective Genetic Algorithm solver for the two conflicting objective functions, namely the total structural weight and the system response variability, for a given stochastic field with a classical Robust Design Optimization formulation. Then, maximum possible variances of the selected designs are computed from the respective maximum values (see Eq. (2)) of the corresponding Variability Response Functions characteristic to these designs. The resulting front is then compared to a new Pareto front in which the second objective function is the maximum possible system variance which can be readily obtained by minimizing the maximum value of the Variability Response Function $\min VRF(\kappa^{\max},\sigma_{ff})$. The former classical RDO front proves to be, as expected, sub-optimal to the VRF-based one since the latter is by definition independent of the probability distribution and the spectral density used to model system's uncertainty. It is mentioned that the generated front and the respective proposed designs are referring to a variety of stochastic fields in contrast to the classical RDO. It is also clarified that the proposed designs are not necessarily optimal when examined under the scope of only one predesignated stochastic field. In the case that an optimization is carried out for a specific correlation structure the resulting design selection will be suboptimal with respect to any other correlation structure.

2. Classical RDO formulation

A general formulation of an optimization problem can be stated as:

$$\left.\begin{array}{llll} \text{optimize}: & f(\mathbf{x}), & & \text{(a)} \\ \text{subject to}: & g_i(\mathbf{x}) \leqslant 0, & i = 1,\dots,I, & \text{(b)} \\ & h_j(\mathbf{x}) = 0, & J = 1,\dots,J, & \text{(c)} \end{array}\right\} \tag{3}$$

where (3b) represents the set of inequality constraints and (3c) the set of equality constraints. In our case objective function $f(\mathbf{x})$ is a bi-objective function taking under consideration total material cost i.e. total structural weight and system variance response. Thus our problem falls into the category of multi-objective optimization with structural cost and robustness of the response being the focus of our design. So the RDO formulation in our example for demonstrative purposes can be stated as follows:

$$\min_{\mathbf{s}\in\mathbf{F}} \quad f = [C(\mathbf{s},\mathbf{x}), \mathrm{var}(\mathbf{u})]^T \tag{4}$$

subjected to deterministic constraints:

$$g_j(\mathbf{x}) \leqslant 0 \quad J = 1,\dots,k \tag{5}$$

where f are the objective functions related to the material cost C and system variance response $var(\mathbf{u})$. Vector $\mathbf{s}$ represents the design variable vectors and $\mathbf{x}$ is the position vector. F is the feasible region where all the deterministic constraint functions g_j are satisfied. It is mentioned here that an alternative second objective function could be selected as opposed to $\mathrm{var}(\mathbf{u})$ i.e. $(\varepsilon[\mathbf{u}] + 3\sigma_{\mathbf{u}})$ where $\sigma_{\mathbf{u}} = \sqrt{\mathrm{var}(\mathbf{u})}$, that would also be a very valid conceptually selection as well. However this would lead to an identical selection of design variables as with the methodology followed in the current work since $\varepsilon(\mathbf{u})$ is almost constant with respect to different *SDF* as shown in [11] and very close to the deterministic displacement $u_{\det}$. Therefore, $\min\{\varepsilon(\mathbf{u}) + 3\sigma_u\}\varepsilon(\mathbf{u}) + 3\min(\sigma_u)$. Apart from this, it is quite common that coefficient of variation *COV* alone is selected as the second counterpart of a bi-objective function in a robust design problem [1,2,6].