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Assessment of polynomial correlated function expansion for high-fidelity structural reliability analysis

Souvik Chakraborty *, Rajib Chowdhury

Department of Civil Engineering, Indian Institute of Technology Roorkee, Roorkee, India

A R T I C L E I N F O

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Accurate prediction of failure probability of a given structural system subjected to parametric uncertainty often leads to a computationally challenging process requiring considerable amount of time. To overcome this issue, it is advantageous to develop non-intrusive model, that approximates the system response and perform all subsequent operations on the developed model. This paper presents a novel non-intrusive algorithm, referred to as polynomial correlated function expansion (PCFE), for high-fidelity reliability analysis. The proposed method expresses the output in a hierarchical order of component functions which facilitate (i) expressing the component functions in term of extended bases, (ii) determination of actual responses at quasi-random sample points, (iii) determination of the unknown coefficients associated with the bases by employing homotopy algorithm and (iv) Monte Carlo simulation. PCFE decouples the stochastic computations and finite element (FE) computations, and consecutively the FE code can be treated as a black box, as in the case of a commercial software. Six numerical problems, involving explicit performance functions and real-life problems described by implicit limit-state functions, have been solved to illustrate the performance of the proposed approach. It is observed that PCFE outperforms the existing approaches.

1. Introduction

The main aim of reliability analysis is the computation of failure probability. Theoretically, this can be easily accomplished by solving a multivariate integral in domains described by failure modes. However in reality, the difficulty arises due to the implicit nature of the limit-state functions. Thus, a detailed finite element modelling of the structure in combination with reliability analysis becomes evident. Even for problems defined by explicit limit-state functions, evaluation of failure probability may be challenging either due to the highly non-linear nature of the limit-state function or due to the irregularities in the problem domain.

The most elegant and widely accepted approach for evaluating failure probability is Monte Carlo simulation (MCS) [1,2]. The foundation of this method is based on an algorithm for generating random numbers. Failure probability is obtained, within a predefined confidence bound, by carrying out large number of simulations at randomly generated sample points and counting the number of responses exceeding threshold limit. A number of modifications to this algorithm, such as directional simulation [3–6], importance sampling (IS) [7–11], subset simulation [12–14] etc., have also been proposed by various researchers. All these approaches are based on the sampling algorithm and therefore, can be categorised as sampling approach (SA). However, SA is computationally burdensome and often becomes impracticable for complex systems having low failure probability.

A totally distinct approach to that discussed above is the non-sampling approach (NSA). Within the framework of this method, one first identify the point, often known as design point or most probable point (MPP), on the limit-state function that is nearest to the origin in the standard normal space. Once the design point has been identified, the limit-state function close to design point is approximated with a truncated Taylor's series expansion. If first order Taylor's series expansion is used to approximate the limit-state function, then the procedure is known as first order reliability method (FORM) [15,16]. Similarly, if a second order Taylor's series is used, it is called second order reliability method (SORM) [17–19]. Once the approximate limit-state function near MPP is obtained, the failure probability is obtained by computing the integral of the approximated function by asymptotic methods. Although this method yields accurate result for weakly non-linear problems [20,21], its accuracy for highly non-linear problem is still questionable. Moreover, when the gradients of the limit-state

* Corresponding author.
E-mail addresses: csouvik41@gmail.com (S. Chakraborty), rajibfce@iitr.ac.in (R. Chowdhury).

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function are unknown, both FORM and SORM becomes computationally burdensome.

Response surface method (RSM) [22–26] is another approach for determining failure probability. This method, in a sense, is a combination of SA and NSA. Initially, one has to determine the responses at some pre-selected sample points in the design space. This step is known as design of experiments (DOE) [27–29]. Once the responses at pre-selected sample points are obtained, an approximate limit-state function is formulated. This approximate limit-state function is the backbone of RSM and often known as surrogate model. It is a well-known fact that the construction of surrogate model is the most crucial step in this procedure. Surrogate models that are popular among researchers for reliability analysis includes but are not limited to high order stochastic response surface method (HO-SRSM) [30], kriging [31–33], polynomial chaos expansion (PCE) [34,35], high dimensional model representation [36–38] and moving least square [39,40].

This paper presents a new surrogate modelling technique, referred to as polynomial correlated function expansion (PCFE), for reliability analysis. Compared to the conventional surrogate models, the proposed approach has several advantages. Firstly, the proposed approach reduces the number of actual function evaluations significantly, making the process computationally efficient. Secondly, PCFE is optimal in Fourier sense. This is because the unknown coefficients associated with the bases are determined by considering the hierarchical orthogonality of the component functions. Furthermore, it is mean-square convergent series as it minimizes the mean square error from truncation after a finite number of cooperative terms. Finally, it is capable of dealing with both correlated and uncorrelated variables without the need for any ad hoc transformations. Although, the proposed method can be invariantly used with both uniformly and non-uniformly distributed sample points, quasi-random sample points have been used in this paper. Out of various available techniques for generating quasi-random sequence, such as Halton sequence [41], Sobol's sequence [42,43] and Faure sequence [44], Sobol's sequence has been used due to its high convergence rate [45].

The paper is organised as follow. After a brief discussion about the use of surrogate modelling in reliability analysis in Section 2, details of the proposed approach is presented in Section 3. Section 4 illustrates the performance of the proposed method in reliability analysis with six problems. Finally, the summary and conclusion of the paper is discussed in Section 5.

2. Surrogate modelling in reliability analysis

Let $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}$ be a N dimensional vector representing the random variables associated with a system and defined in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ where, $\Omega \rightarrow \mathbb{R}^N$. If $g(\mathbf{x})$ represents the limit-state function such that $g(\mathbf{x}) \geqslant 0$ denotes the safe region and $g(\mathbf{x}) < 0$ is the failure zone, probability of failure P_f is defined as

$$P_f = \int_{g(\mathbf{x})<0} f(\mathbf{x}) d\mathbf{x} \tag{1}$$

where, $f(\mathbf{x})$ denotes the joint probability distribution of $\mathbf{x}$. From Eq. (1), it is evident that knowledge of $g(\mathbf{x})$ is essential for determination of P_f. However, explicit form for the limit-state function $g(\mathbf{x})$ is often unknown for practical problems. In surrogate modelling, we generate an approximate limit-state function $\hat{g}(\mathbf{x})$ based on actual responses at selected sample points and by minimizing the error between the actual and approximate model in some statistical sense. Thus, failure probability P_f^{SM} in surrogate modelling approach is defined as

$$P_f^{SM} = \int_{\hat{g}(\mathbf{x})<0} f(\mathbf{x}) d\mathbf{x} \tag{2}$$

It is apparent that construction of $\hat{g}(\mathbf{x})$ is of vast importance in surrogate modelling for accurate prediction of failure probability P_f. Once $\hat{g}(\mathbf{x})$ is obtained, P_f may be obtained by performing FORM/SORM/IS/MCS on the explicit form of $\hat{g}(\mathbf{x})$.

3. Polynomial correlated function expansion (PCFE)

Let, $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}$ be an N dimensional vector, representing the input variables of a structural system. It is quite logical to express the output response $g(\mathbf{x})$ as a finite series as [46,47,49–51]

$$g(\mathbf{x}) = g_0 + \sum_{k=1}^{N} \sum_{i_1<i_2<\cdots<i_k}^{N} g_{i_1 i_2 \ldots i_k}(x_{i_1}, x_{i_2}, \ldots, x_{i_k}) \tag{3}$$

where, g_0 is constant and represents the mean response.

Suppose, two subspace R and B in Hilbert space is spanned by basis $\{r_1, r_2, \ldots, r_l\}$ and $\{b_1, b_2, \ldots, b_m\}$ respectively. Now if, (i) $B \supset R$ and (ii) $B = R \oplus R^{\perp}$ where, $R^{\perp}$ is the orthogonal complement subspace of R in B, we term B as extended basis and R as non-extended basis [48]. Now if ψ represents polynomial basis for $\mathbf{x}$, Eq. (3) can be rewritten, in terms of extended bases, as

$$g(\mathbf{x}) = g_0 + \sum_{k=1}^{N} \sum_{i_1<i_2<\cdots<i_k} \left(\sum_{r=1}^{k} \left(\sum_{m_1} \sum_{m_2} \cdots \sum_{m_r} \alpha_{m_1 m_2 \cdots m_r}^{(i_1 i_2 \cdots i_k) i_r} \psi_{m_1}^{i_1}(x_{i_1}) \psi_{m_2}^{i_2}(x_{i_2}) \cdots \psi_{m_r}^{i_r}(x_{i_r}) \right) \right) \tag{4}$$

where, α denote unknown coefficients associated with the bases. Eq. (4) represents the basic form of PCFE.

Definition 1. The terms in Eq. (4) reflecting independent effect only are termed as first-order component function and denoted as $g_i(x_i)$. Similarly, the terms reflecting bivariate effect are termed as second-order component function and denoted by $g_{ij}(x_i, x_j)$. The constant g_0 is termed as zeroth order component function, and can be treated as average response at all the selected sample points.

Definition 2. The expression obtained by substituting $N = S$ in Eq. (4) is termed as Sth order PCFE expression. An Sth order PCFE consists of all the component functions up to Sth order, i.e., while first-order PCFE consists zeroth and first order component functions, a second-order PCFE consists zeroth, first and second order component functions. Therefore, adding all the Sth order component functions to an existing $(S-1)$th order PCFE would yield the Sth order PCFE expression.

3.1. Solution strategy

An essential condition, associated with Eq. (4), is the hierarchical orthogonality of the component functions. This require a higher order component function to be orthogonal with all the lower order component functions. To determine the unknown coefficients $\boldsymbol{\alpha}$ while satisfying the orthogonality criteria, a homotopy algorithm [52–54] is employed.

Rewriting Eq. (4) in matrix form

$$\boldsymbol{\Psi}\boldsymbol{\alpha} = \mathbf{d} \tag{5}$$

where, $\boldsymbol{\Psi}$ and $\boldsymbol{\alpha}$ consists of the basis functions and unknown coefficients respectively. If $\mathbf{g} = \{g_1, g_2, \ldots, g_{N_s}\}^T$ be the observed responses at N_s set of sample points and $\bar{\mathbf{g}} = \{g_0, g_0, \ldots, g_0\}^T$ be the mean response vector, then $\mathbf{d} = \mathbf{g} - \bar{\mathbf{g}}$. Pre-multiplying Eq. (5) by $\boldsymbol{\Psi}^T$, we obtain