Structural Safety 59 (2016) 32–41

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Efficient estimation of the peak factor for the stochastic characterization of structural response to non-stationary ground motions

Ignacio Ferrer [a,*], Carlos R. Sánchez-Carratalá [b]

[a] Department of Mechanics of Continuous Media and Structural Analysis, School of Design Engineering, Universidad Politécnica de Valencia, Camino de Vera s/n, 46022 Valencia, Spain
[b] Department of Mechanics of Continuous Media and Structural Analysis, School of Civil Engineering, Universidad Politécnica de Valencia, Camino de Vera s/n, 46022 Valencia, Spain

ARTICLE INFO

Article history:
Received 23 March 2015
Received in revised form 1 November 2015
Accepted 13 November 2015
Available online 23 December 2015

Keywords:
Non-stationary stochastic processes
Peak factor
Spectral peakedness factor
Performance-based design
Seismological models
Equivalent stationary duration

ABSTRACT

The determination of the peak factor is quite cumbersome in the case of a non-stationary process, due to the necessity of calculating the non-geometric spectral moments of the process and obtaining the threshold level of the first-passage problem corresponding to a certain non-exceedance probability value. In this paper, an efficient approximate new method is presented for practical applications in Earthquake Engineering. The so-called Equivalent Oscillator and Response method consists of transforming the original oscillator excited by a uniformly modulated non-stationary colored noise into an equivalent oscillator excited by a uniformly modulated non-stationary white noise, in such a way that the responses of both oscillators have similar values for the spectral parameters that affect the calculation of the peak factor. A new dimensionless spectral peakedness factor is introduced in order to ensure the matching of the spectral bandwidth of both oscillators. A numerical application of the new method, based on a non-stationary seismological model for the ground accelerations process, is carried out, showing the reliability and robustness of the new method for efficiently and accurately estimating the peak factor of the non-stationary structural response.

1. Introduction

In the last decades, the estimation of the peak factor has been a very active research field in Engineering Seismology due to the growing interest in performance-based earthquake resistant design [1]. In fact, several methods of estimating the maximum value of non-stationary stochastic processes have been developed in the context of Random Vibration Theory [2–5]. The computation of the maximum response is relatively easy if the oscillator response is considered as a stationary process, because explicit expressions can be derived for the peak factor. However, if the oscillator response is assumed to be non-stationary, computing the maximum response becomes a lengthy procedure, especially due to the following issues: several convolution products must be carried out to determine the spectral moments of the structural response process, and a non-linear integral equation must be solved to evaluate the maximum response. This paper proposes an efficient approximate method—called Equivalent Oscillator Response method—of estimating the peak structural response of a linear SDOF structural system subjected to a non-stationary ground motion, that eliminates the shortcomings indicated above and reduces drastically the calculation time. The basis of the stochastic processes and Random Vibration Theory are briefly introduced in order to present the notation and to clarify the hypotheses used in the method. Later on, the new method is developed and applied in a numerical analysis based on a ground seismological model.

2. Structural response

2.1. Stochastic excitation process

Let $\{a_g(t)\}$ be a non-stationary stochastic process representing the ground accelerations produced by an earthquake at a specific location over time. A zero-mean ground accelerations process will be assumed hereafter without loss of generality. In this paper, the evolutionary spectral representation proposed by Priestley [6] with a frequency-independent modulation of the acceleration amplitudes is assumed, commonly known as uniformly modulated process. Both hypotheses are very usual in Earthquake Engineering. Then, the realizations of the stochastic process $\{a_g(t)\}$ can be expressed as:

$$a_g(t) = I_{ag,us;ag}(t)a_{g,us}(t) \quad (1)$$

* Corresponding author.
E-mail addresses: igferbal@mes.upv.es (I. Ferrer), csanchez@mes.upv.es (C.R. Sánchez-Carratalá).

http://dx.doi.org/10.1016/j.strusafe.2015.11.004
0167-4730/

where $I_{ag,us;ag}(t)$ is a deterministic intensity function with a slow variation over time, and $a_{g,us}(t)$ are the realizations of the underlying stationary process $\{a_{g,us}(t)\}$ which has a spectral representation of the form:

$$a_{g,us}(t) = \int_{-\infty}^{\infty} e^{j2\pi ft} d\widetilde{Z}_{ag,us}(f) \quad (2)$$

where $j = \sqrt{-1}$ is the imaginary unit, f is the cyclic frequency, the tilde over a function is used to highlight the fact that the function is complex-valued and $\{\widetilde{Z}_{ag,us}(f)\}$ is a complex-valued stationary random process with orthogonal increments, i.e., $E\left[d\widetilde{Z}^*_{ag,us}(f_k)\, d\widetilde{Z}_{ag,us}(f_l)\right] = G_{ag,us}(f_k)\,\delta(f_k - f_l)\, df_k\, df_l$, being $E[\cdot]$ an operator that gives the mathematical expectation of the argument, the superscript "$*$" means complex conjugate, $\delta(\cdot)$ is the Dirac delta function, and $G_{ag,us}(f)$ is the two-sided variance spectral density function. Note that the subscript "$ag, us; ag$" in the intensity function is used to denote that this function transforms $a_{g,us}(t)$ into $a_g(t)$.

2.2. Stochastic response process

The displacement $d(t)$ of a viscously damped linear time-invariant SDOF oscillator, of natural cyclic frequency f_n, excited by a time series of ground accelerations $a_g(t)$ and with null initial conditions—i.e., $d(t) = 0, \dot{d}(t) = 0$—is given by replacing Eqs. (1) and (2) into the Duhamel integral. Then, the realizations of $\{d(t)\}$ can be computed as follows:

$$d(t) = \int_{-\infty}^{\infty} \widetilde{I}_{ag,us;d}(f,t)\, e^{j2\pi ft} d\widetilde{Z}_{ag,us}(f) \quad (3)$$

where $\widetilde{I}_{ag,us;d}(f,t)$ is a complex-valued intensity function whose expression is the following:

$$\widetilde{I}_{ag,us;d}(f,t) = \int_{-\infty}^{\infty} h_{ag;d}(\tau)\, I_{ag,us;ag}(t-\tau)\, e^{-j2\pi f\tau} d\tau \quad (4)$$

where $h_{ag;d}(t)$ is the impulse response function of the oscillator, which has the following expression:

$$h_{ag;d}(t) = -\frac{1}{2\pi f_d}\, e^{-\xi 2\pi f_n t} \sin(2\pi f_d t) \quad \text{for } t \geqslant 0 \quad (5)$$

where ξ is the damping ratio and f_d is the damped cyclic frequency. Note that the subscript "$ag; d$" in the impulse response function is used to denote that this function transforms $a_g(t)$ into $d(t)$. In the frequency domain, the impulse response function becomes the well known transfer function of the viscously damped oscillator:

$$\widetilde{H}_{ag;d}(f) = -\frac{1}{(2\pi f_n)^2}\, \frac{1}{(1-\beta^2) + j2\xi\beta} \quad (6)$$

where β is the frequency ratio, given by $\beta = f/f_n$.

Taking into consideration that $I_{ag,us;ag}(t)$ is an intensity function, then $\widetilde{I}_{ag,us;d}(f,t)$—which is the convolution of $\widetilde{I}_{ag,us;ag}(f,t)$ and a phasor with modulus equal to the impulse response function of the filter—will also have a slow variation over time, so that it can also be considered as an intensity function. Hence, if the seismic action $\{a_g(t)\}$ is modeled by an evolutionary stochastic process, it can be stated that the oscillator response process $\{d(t)\}$ is also evolutionary, and is modulated by the intensity function defined in Eq. (4). It should be emphasized that this intensity function operates on the underlying stationary process of the ground accelerations, $\{a_{g,us}(t)\}$, and not on the underlying stationary process of the displacements response, $\{d_{us}(t)\}$.

2.3. Variance and cross-correlation coefficient of the response process

The peak factor of a non-stationary process is directly expressed in terms of the following four functions: the variance functions of the displacements and velocities of the response process—i.e., $\sigma_d^2(t)$ and $\sigma_v^2(t)$—; the normalized zero-time lag cross-covariance function between the displacements and velocities of the response process—i.e., $\kappa_{dv}^2(t)$—, also known as cross-correlation coefficient; and the spectral bandwidth function of the displacements of the response process—i.e., $q_d(t)$—. According to the developments carried out by Michaelov et al. [3,4], these functions can be computed as follows:

$$\begin{aligned} \sigma_d^2(t) &= (m_{0,0})_d(t) & \kappa_{dv}^2(t) &= -\frac{\mathrm{Im}\left[(\tilde{m}_{0,1})_d(t)\right]}{\sqrt{(m_{0,0})_d(t)}\,\sqrt{(m_{1,1})_d(t)}} \\ \sigma_v^2(t) &= (2\pi)^2 (m_{1,1})_d(t) & q_d(t) &= \sqrt{1 - \frac{\mathrm{Re}^2\left[(\tilde{m}_{0,1})_d(t)\right]}{(m_{0,0})_d(t)\,(m_{1,1})_d(t)}} \end{aligned} \quad (7)$$

where $(m_{0,0})_d(t)$, $(m_{1,1})_d(t)$ and $(\tilde{m}_{0,1})_d(t)$ are the spectral moments of order (0,0), (1,1) and (0,1), respectively, and $\mathrm{Re}[\cdot]$ and $\mathrm{Im}[\cdot]$ are operators that give the real and the imaginary part of the argument, respectively. It must be noted that the spectral moments used here are calculated from the variance spectrum of the response process in terms of the cyclic frequency, and not of the circular frequency.

Two approaches for the definition of the spectral moments can be found in the scientific literature: the geometric approach proposed by Vanmarcke [7], and the non-geometric one introduced by Di Paola [8]. The non-geometric definition of non-stationary spectral moments will be used in the present paper. Michaelov et al. [3,4] make a comprehensive review of this approach, and apply it to the calculation of non-geometric spectral moments—also known as spectral characteristics—of evolutionary processes, obtaining the following expressions:

$$\begin{aligned} (m_{0,0})_d(t) &= 2\int_0^\infty \left[I^{(0)}_{ag,us;d}(f,t)\right]^2 G_{ag,us}(f)\, df \\ (m_{1,1})_d(t) &= \frac{2}{(2\pi)^2}\int_0^\infty \left[I^{(1)}_{ag,us;d}(f,t)\right]^2 G_{ag,us}(f)\, df \\ (\tilde{m}_{0,1})_d(t) &= -\frac{2j}{2\pi}\int_0^\infty \widetilde{I}^{(0)*}_{ag,us;d}(f,t)\, \widetilde{I}^{(1)}_{ag,us;d}(f,t)\, G_{ag,us}(f)\, df \end{aligned} \quad (8)$$

where $\widetilde{I}^{(k)}_{ag,us;d}$ has the following recursive expression:

$$\widetilde{I}^{(k)}_{ag,us;d}(f,t) = \frac{\partial \widetilde{I}^{(k-1)}_{ag,us;d}(f,t)}{\partial t} + j2\pi f \widetilde{I}^{(k-1)}_{ag,us;d}(f,t) \quad (9)$$

Note that for the stationary case—$\widetilde{I}^{(0)}_{ag,us;d}(f,t) = 1\ \forall t$—the spectral moments $(m_{k,l})_d(t)$ of Eq. (8) become $(m_{k+l})_d$. The computation of the spectral moments of the response process from Eq. (8) is a very time-consuming procedure due to the calculation of the complex-valued intensity function $\widetilde{I}^{(0)}_{ag,us;d}(f,t)$, that requires making a time convolution product for each frequency f and for each time t, according to Eq. (4). The function $\widetilde{I}^{(1)}_{ag,us;d}(f,t)$ is then obtained with Eq. (9). For instance, in the case of a uniformly modulated input process, if the variance spectrum $G_{ag,us}(f)$ is defined with a sampling frequency interval $\Delta f = 0.02$ Hz up to a frequency of 20 Hz, and the frequency-independent intensity function $I_{ag,us;ag}(t)$ is defined with a sampling time interval of $\Delta t = 0.01$ s up to a time of 25 s, then a total number of 1000×2500 convolution products have to be calculated to obtain $\widetilde{I}_{ag,us;d}(f,t)$ as a discrete function represented by a 1000x2500 matrix.

If the underlying stationary process of the ground motion $\{a_{g,us}(t)\}$ is a white noise process $\{w_{us}(t)\}$, then the two-sided variance spectrum is a constant value $G_{w,us}$. In this case, Eq. (8) transforms into the following expressions derived by Michaelov et al. [4]: