Structural Safety 54 (2015) 1–9

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

An efficient method for generation of uniform support vector and its application in structural failure function fitting

Youbao Jiang *, Jun Luo, Guoyu Liao, Yulai Zhao, Jianren Zhang

School of Civil Engineering and Architecture, Changsha University of Science and Technology, Changsha 410004, China

ARTICLE INFO

Article history:
Received 11 February 2014
Received in revised form 28 June 2014
Accepted 28 December 2014
Available online 30 January 2015

Keywords:
Structural reliability
Support vector
Sample pair
Uniform design
Failure function

ABSTRACT

The current response surface methods based on support vector usually need a large number of samples to fit an implicit structural failure function. To overcome this shortcoming, an efficient method for generation of uniform support vector is proposed. It is based on the features that support vectors are composed of failure samples and safe samples close to the limit state surface. The main steps are: (1) use the uniform design method to generate initial samples; (2) transform each obtained initial sample into a uniform sample pair based on the safe load and failure load close to the limit load. A main advantage of this method is that it can increase the proportion of support vectors to the whole samples and uniformity of support vectors in space dramatically and it requires less samples in function fitting. Besides, it can be applied to function fittings of large structures under ultimate limit state, where multiple failure modes may be enveloped. The proposed method as well as the relevant techniques of data normalization and parameters optimization of kernel function model of support vector machine, is used in the structural failure function fitting. Numerical examples show that this method can achieve a good fitting of implicit failure function, and the reliability results are accurate, too.

1. Introduction

For large structures, the ultimate bearing capacities and the interested node displacements are usually obtained by finite element analysis (FEA) rather than by analytical formulas, thus its failure function is implicit, which leads to some difficulties in solution of reliability index. In this case, the response surface method is one of the main methods to solve such difficulties due to its convenience in combination with all kinds of FEA software.

Faravelli and Bigi [1,2] earlier discussed a stochastic finite element method based on response surface approximation to analyze the reliability of structural and mechanical systems whose geometrical and material properties have spatial random variability. Breitung and Faravelli [3,4] also investigated response surface methods and asymptotic approximations in structural reliability assessment, and proposed a log-likelihood maximization approach for reliability assessment, which can be used for dependent random variables. The main advantage of the method of log-likelihood maximization is the fact that the response-surface iterative scheme can be used in the original space of the random variables and appropriate space transformations are not preliminarily required.

Based on these approaches above, the structural reliability computation can be carried out whether the random variable is dependent or not. Thus, the interests can be mainly focused on how to achieve an accurate and efficient fitting of implicit structural failure function with the response surface method.

Bucher and Bourgund [5] studied a new adaptive interpolation scheme of updating polynomial to increase the efficiency and accuracy of the response surface method in reliability calculation. Rajashekhar and Ellingwood [6] stated that the accuracy of a response surface depends on the characteristics of the limit state being explored and hence one cycle of updating may not always be sufficient, and investigated some methods to ensure that the response surface fits the actual limit state in the region of maximum likelihood.

For a large structures, the actual failure function would be usually a complex one. Thus, the problem how to use the response surface method to perform an accurate fitting of a complex function is still worth being studied. Many scholars have noticed this problem and proposed many different measures to improve the efficiency and accuracy of the response surface method. These improved methods include the ones using quadratic polynomial [7–10] and the ones using artificial neural network [11,12]. However, these methods [7–12] are mainly based on the principle of empirical risk minimization (i.e., fitting residual minimization). Thus, the fitting accuracy is affected largely by the number of samples. Besides, its generalization error would also increase as the number of variables increases [13].

* Corresponding author.

http://dx.doi.org/10.1016/j.strusafe.2014.12.004
0167-4730/

Based on the statistical learning theory, an optimum way to minimize the deviation between the input and output of a learning machine is following the principle of structural risk minimization. Support vector machine (SVM) is the right way to follow this principle. It has been widely used in reliability research fields due to its superior performance in dealing with small sample size problem.

Dai et al. [14] proposed an improved approach combining the SVM with the important sampling technique to perform reliability analyses. Numerical calculations indicate that this improved approach is efficient. Li et al. [15] and Moura et al. [16] compared the efficiency and accuracy of the response surface method based on SVM and other conventional response surface methods through the reliability calculations of several examples. It indicates that the method based on SVM can be as accurate as those based on artificial neural network and quadratic polynomial; however, the method based on SVM needs less samples. Furthermore, Wang et al. [17] proposed a method that combines the least squares SVM with the Monte Carlo simulation to analyze the foundation settlement reliability. Since the solution of least squares SVM is quick, it can overcome the defects of the conventional SVM methods which would be more time-consuming to train samples.

Actually, the efficiency and accuracy of the response surface method depends on not only the fitting models (e.g., SVM, quadratic polynomial), but also the distributions of samples in space. Fang and Wang [18] proposed a uniform design method to achieve a better function fitting, by which samples can be distributed as uniformly as possible. The uniform design method has been widely applied to the response surface fittings. For example, Lü et al. [19] combined the uniform design method with the artificial neural network and obtained a good fitting in the probabilistic ground-support interaction analysis of deep rock excavation. Li et al. [20] also applied such method successfully to a SVM model selection in face recognition.

Notice that the SVM technique only uses the support vectors rather than any other sample to fit a function, thus the fitting parameters are only dependent of support vectors. However, the methods [14–17] do not pay any special attention to this feature, resulting that the support vectors only take up a minor proportion of the whole samples. Thus, the required number of samples is still large when such methods are applied to function fittings. Besides, even though the whole samples generated by the methods [19,20] can be distributed uniformly, the distributions of support vectors may be less uniform. It indicates that the current response surface methods based on SVM still need to be improved in efficiency and accuracy.

This study tries to propose an efficient method for generation of uniform support vector to fulfill this demand, which is mainly based on the sample pair generating technique and the uniform design method. The proposed method as well as the *K*-fold cross-validation method, which deals with parameters optimization of kernel function model of SVM, is used in the structural failure function fitting. The computing efficiency and accuracy is also studied for the fitting method based on uniform support vector.

2. Support vector machine

In this section, some basic concepts of support vector machine are briefly introduced. More details of support vector machine can be found in [21,22].

2.1. Optimum linear classifier

Given a set of n training samples $(\boldsymbol{x}_i, y_i)$ ($i = 1, 2, \ldots, n$) with binary outputs $y \in \{+1, -1\}$ corresponding to the two classes. Assume that this set is linearly separable, as shown in Fig. 1, and the decision hyper-plane is expressed as $G(\boldsymbol{x}) = \boldsymbol{w} \cdot \boldsymbol{x} + \boldsymbol{b}$, which separates the training samples (i.e., which obeys $|G(\boldsymbol{x})| \geqslant 1$ for all $\boldsymbol{x}_i$). Thus, the margin is $2/\|\boldsymbol{w}\|$, and the optimum linear classifier is solved and given by

$$G(\boldsymbol{x}) = \mathrm{sgn}\left[\sum_{i=1}^{n} \alpha_i^* y_i (\boldsymbol{x}_i \cdot \boldsymbol{x}) + b^*\right] \tag{1}$$

where $\mathrm{sgn}(\cdot)$ denotes the sign function; $(\boldsymbol{x}_i \cdot \boldsymbol{x})$ denotes the inner product operation; α_i^* and b^* are two relevant parameters to define the optimum linear classifier. For most samples, $\alpha_i^* = 0$. By comparison, for support vectors, $\alpha_i^* \neq 0$.

However, when a sample set is not linearly separable, a relaxation parameter ξ_i and a penalty parameter C is introduced to obtain an optimum linear classifier. The corresponding equation is given by

$$\begin{aligned} &\min \quad \frac{1}{2}\|\boldsymbol{w}\|^2 + C\xi_i \\ &s.t. \quad |\boldsymbol{w} \cdot \boldsymbol{x} + b| \geqslant 1 - \xi_i \end{aligned} \tag{2}$$

Its solution can also be found in [21].

2.2. Kernel function models

For a nonlinear classifier, a kernel function instead of the inner product operation is introduced. Let $\mathrm{Kf}(\boldsymbol{x}_i \cdot \boldsymbol{x})$ denote the kernel function, then Eq. (1) becomes

$$G(\boldsymbol{x}) = \mathrm{sgn}\left[\sum_{i=1}^{n} \alpha_i^* y_i \mathrm{Kf}(\boldsymbol{x}_i \cdot \boldsymbol{x}) + b^*\right] \tag{3}$$

The common kernel function includes three types: Gauss, polynomial and sigmoidal kernel function. In this paper, the polynomial kernel function is selected and given by

$$\mathrm{Kf}(\boldsymbol{x}_i \cdot \boldsymbol{x}) = [\gamma(\boldsymbol{x}_i \cdot \boldsymbol{x}) + 1]^d \tag{4}$$

It is reported that when d is less than 5, the fitting accuracy would be better in the previous studies. Besides, other parameters (e.g., γ, penalty parameter C) also play an important role to establish a SVM model. Obviously, the values of such parameters would greatly affect the generalization of the obtained SVM model. Thus, some effective measures for parameters optimization are introduced in the following sections.

2.3. Measures for parameters optimization of kernel function model

Recently, the cross-validation method is one of the mainly used methods to complete parameters optimization of kernel function model [22]. It includes *K*-fold cross-validation method, Hold-Out method and leave-one-out cross-validation method. Here, the *K*-fold cross-validation method is selected.

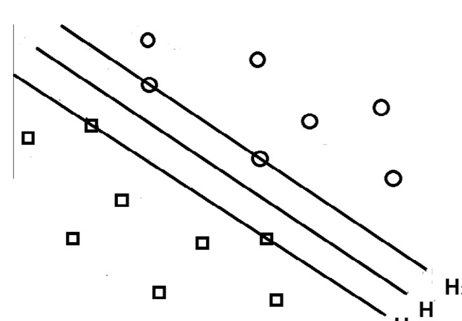

Fig. 1. Explanation of linear classification surface.