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Alternative environmental contours for structural reliability analysis

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ABSTRACT

This paper presents alternative methods for constructing environmental contours for probabilistic structural reliability analysis of structures exposed to environmental forces such as wind and waves. For such structures, it is important to determine the environmental loads to apply in structural reliability calculations and structural design. The environmental contour concept is an effective, risk-based approach in establishing such design conditions. Traditionally, such contours are established by way of a Rosenblatt transformation from the environmental parameter space to a standard normal space, which introduces uncertainties and may lead to biased results. The proposed alternative approach, however, eliminates the need for such transformations and established environmental contours based on direct Monte Carlo sampling from the joint distribution of the relevant environmental parameters. In this paper, three alternative implementations of the proposed generic approach will be outlined.

1. Introduction and background

Probabilistic structural reliability analysis is performed to ensure that a structure is able to withstand the required design loads. A realistic description of the environmental loads and structural response is a crucial prerequisite for structural reliability analysis of structures exposed to environmental forces. In principle, full long-term response analyses should be considered [28], but this is normally very time-consuming and computational intensive. The concept of environmental contours is an efficient method of estimating extreme conditions as basis for design [38,13], and is widely used in marine structural design (see e.g. [2,11,18,22,8]). This approach is also recommended by DNV-GL [9]. The main idea is to use the Rosenblatt transformation in order to transform the environmental variables into independent standard normally distributed variables and identify a sphere with desired radius in this transformed space. Environmental contours are then constructed by re-transforming the sphere back to the original space. This approach is closely related to the FORM-approximation (First Order Reliability Method), where the failure boundary in the transformed space is approximated by a hyperplane at the design point.

Transformations between the original space and the normal space will typically be *non-linear*. This makes the interpretation of the resulting contours less straightforward. Potential problems caused by this are discussed in detail in Huseby et al. [15]. A brief introduction to probabilistic structural design and the traditional approach to environmental contours is also given in Huseby et al. [15].

In the present paper we will focus our attention on methods where the contours are constructed directly in the original space, utilizing Monte Carlo simulations of the joint environmental model. This yields a more straightforward interpretation of the contours. Another advantage is a more flexible framework for establishing environmental contours, which for example simplifies the inclusion of effects such as future projections of the wave climate related to climatic change [31]. Other examples of applications of Monte Carlo methods in structural reliability analysis are presented in e.g. Næss et al. [25], Næss et al. [26], Zhang et al. [39], Juncher Jensen et al. [20].

Safety regulations and construction standards should ensure that structures are able to withstand the external loads and avoid structural failure. In probabilistic structural design, structural reliability analysis forms the basis for rule development. Such safety rules and standards should guide the individual design by restricting the allowable design space, but structural reliability analyses may also be used for individual designs. The main idea is to make sure that the reliability of the structure is sufficient, corresponding

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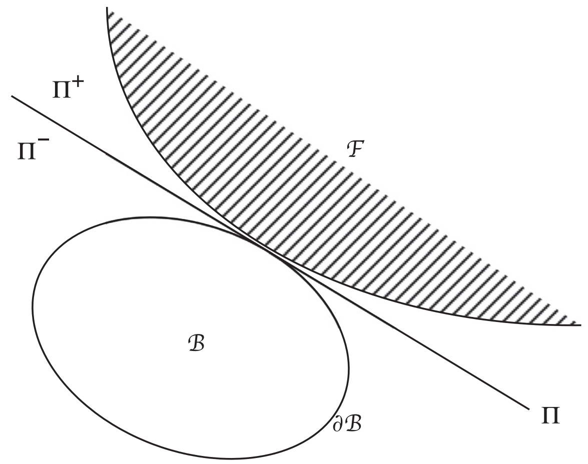

Fig. 1. For any convex failure region $\mathcal{F}$ such that $\mathcal{F} \cap \mathcal{B} = \emptyset$ there exists a hyperplane Π supporting $\mathcal{B}$ such that $\mathcal{F} \subseteq \Pi^+$.

to a maximum allowable failure probability. The reliability of marine structures implicit in safety rules is discussed in Bitner-Gregersen et al. [6].

In the following, the proposed approach to environmental contours using direct Monte Carlo simulations will be briefly reviewed and three specific methods for identifying the contours are presented (Section 3). It is noted that the first method has been discussed previously in Huseby et al. [15], but it is included herein for completeness. Methods 2 and 3 are not previously published but Method 2 has been briefly presented in Huseby et al. [14]. In Section 5, the methods are illustrated by case studies where environmental contours for significant wave height and zero up-crossing wave period are calculated. Furthermore, environmental contours for significant wave height and wave steepness are presented, illustrating the need, in some cases, for a standardization of the variables when they are numerically very different. Section 6 discusses some important issues and compares the proposed alternative method with the traditional one and also presents a brief inter-comparison of the three methods. Finally, a summary is provided in Section 7. It is noted that in this paper the methods are illustrated in two dimensions only, but that, in general, they are easily extended to higher dimensions. The only requirement is that the joint distribution of stochastic input parameters, henceforth referred to as the environmental model, is possible to simulate from.

2. Environmental contours

Environmental contours are defined in various ways in the literature [21,23,12]. In this paper, however, the same understanding of environmental contours as used in Huseby et al. [15] will be held: Let $\mathbf{X}$ be a vector of environmental variables with possible values in the set $\mathcal{X} \subseteq \mathbb{R}^n$ and assume that the distribution of $\mathbf{X}$ is absolutely continuous with respect to the Lebesgues measure in $\mathbb{R}^n$. Moreover, let $P_e \in (0, 0.5)$ be a given exceedence probability. The objective is to identify a convex set $\mathcal{B} \subset \mathcal{X}$ such that for every supporting hyperplane[1] Π of $\mathcal{B}$, we have $P[\mathbf{X} \in \Pi^+] = P_e$, where Π^+ denotes the halfspace bounded by the hyperplane Π and not containing $\mathcal{B}$. We also introduce Π^- which denotes the halfspace complementary to Π^+. Thus, $\mathcal{B} \subseteq \Pi^-$. The resulting environmental contour is the boundary of the set $\mathcal{B}$ and denoted $\partial\mathcal{B}$. Whenever such a set $\mathcal{B}$ can be found, safely designed structures can easily be identified.

[1] A hyperplane Π is a supporting hyperplane of a convex set $\mathcal{B}$ if $\mathcal{B}$ is entirely contained in one of the two closed half-spaces determined by Π and $\mathcal{B}$ has at least one boundary-point on Π.

Fig. 1 illustrates how this can be done. For a given failure region $\mathcal{F}$ we may check that the corresponding failure probability, $P[\mathbf{X} \in \mathcal{F}]$, is bounded by the exceedence probability, P_e, simply by verifying that $\mathcal{F}$ is a convex set such that $\mathcal{F} \cap \mathcal{B} = \emptyset$. If this is the case, it follows by standard convexity theory that there will always exist a hyperplane Π such that Π supports $\mathcal{B}$ and such that $\mathcal{F} \subseteq \Pi^+$. Hence, obviously the failure probability $P[\mathbf{X} \in \mathcal{F}]$ is less than or equal to $P[\mathbf{X} \in \Pi^+] = P_e$, as stated.

In a real-life application the exact shape of the failure region is usually not determined in detail. Instead one investigates the states along the contour $\partial\mathcal{B}$, or at least the most extreme ones, and verifies that the structure does not fail for any of these states. Given that this holds true, one typically assumes that the structure is safe for all the interior states in $\mathcal{B}$ as well. Thus, one may conclude that the failure region does not intersect with $\mathcal{B}$. It then only remains to argue that it is reasonable to assume that $\mathcal{F}$ is convex. That is, if $\mathbf{X}_1$ and $\mathbf{X}_2$ are two arbitrary failure states, one must argue that $\alpha\mathbf{X}_1 + (1-\alpha)\mathbf{X}_2$, where $\alpha \in [0, 1]$, is a failure state as well. Even without exact knowledge about $\mathcal{F}$, this may often be a reasonable assumption.

It is of interest to compare the direct simulation based approach to environmental contours constructed using the traditional approach, i.e., by using the Rosenblatt transformation. The Rosenblatt transformation is a transformation $T : \mathcal{X} \to \mathbb{R}^n$ such that $\mathbf{Y} = T(\mathbf{Y})$ is a vector of n independent standard normally distributed variables. A contour for $\mathbf{Y}$ with the desired exceedence probability, P_e can easily be constructed analytically, and the resulting contour in $\mathcal{X}$ is obtained by using the inverse transformation of T. As for the direct simulation approach, one proceeds by verifying that the structure does not fail for any of the states along the contour and thus concludes that the failure region does not intersect with the region surrounded by the contour. However, in order to verify that the failure probability, $P[\mathbf{X} \in \mathcal{F}]$, is bounded by the exceedence probability, P_e, one must in this case argue that the corresponding *transformed failure region* $T(\mathcal{F})$ is convex. That is, if $\mathbf{X}_1$ and $\mathbf{X}_2$ are two arbitrary failure states, one must argue that $T^{-1}(\alpha T(\mathbf{X}_1) + (1-\alpha)T(\mathbf{X}_2))$, where $\alpha \in [0, 1]$, is a failure state as well. Since the transformation T depends on the joint probability distribution of $\mathbf{X}$ and not just on the physical properties of the structure, it is much more difficult to justify such an assumption without explicitly determining the region $\mathcal{F}$ in detail. Still in well-behaved cases where the transformation T is not strongly non-linear, the transformation approach will produce contours which are close to the contours constructed using a direct simulation based approach.

Obviously, the properties of the set $\mathcal{B}$ ultimately depends on the probability distribution of the vector $\mathbf{X}$. If $\mathcal{B}$ has the property mentioned above, we say that $\mathbf{X}$ admits a P_e-contour. From the above definition we see that the construction of $\mathcal{B}$ is strongly linked to hyperplanes Π with the property that $P[\mathbf{X} \in \Pi^+] = P_e$. We will refer to such hyperplanes as P_e-exceedence hyperplanes, and we denote by $\mathcal{P}(P_e)$ the family of all P_e-exceedence hyperplanes. The following basic result states that P_e-exceedence hyperplanes actually exist.

Proposition 2.1. *Let Π be an arbitrary hyperplane in $\mathbb{R}^n$. Then there exists a P_e-exceedence hyperplane $\tilde{\Pi}$ which is parallel to Π.*

Proof. Let $\mathbf{c} \in \mathbb{R}^n$ be a orthogonal vector to Π, and introduce the random variable $Y = \mathbf{cX}$. Since we have assumed that the distribution of $\mathbf{X}$ is absolutely continuous with respect to the Lebesgues measure in $\mathbb{R}^n$, it follows that the cumulative distribution function of Y, $F_Y(y)$ is a continuous function. Hence, there exists a number y_e such that $P[Y > y_e] = (1 - F_Y(y_e)) = P_e$. We then define the hyperplane $\tilde{\Pi}$ as:

$$\tilde{\Pi} = \{\mathbf{x} : \mathbf{cx} = y_e\}.$$

Moreover, let the halfspace $\tilde{\Pi}^+$ be: