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Parameter sensitivity of system reliability using sequential compounding method

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ABSTRACT

Computation of sensitivities of the 'system' failure probability with respect to various parameters is essential in reliability based design optimization (RBDO) and uncertainty/risk management of a complex engineering system. The system failure event is defined as a logical function of multiple component events representing failure modes, locations or time points. Recently, the sequential compounding method (SCM) was developed for efficient calculations of the probabilities of large-size, general system events for a wide range of correlation properties. To facilitate the use of SCM in RBDO and uncertainty/risk management under a constraint on the system failure probability, a method, termed as Chun–Song–Paulino (CSP) method, is developed in this paper to compute parameter sensitivities of system failure probability using SCM. For a parallel or series system, the derivative of the system failure probability with respect to the reliability index is analytically derived at the last step of the sequential compounding. For a general system, the sensitivity of the probability of the set involving the component of interest and the sensitivity of the system failure probability with respect to the super-component representing the set are computed respectively using the CSP method and combined by the chain-rule. The CSP method is illustrated by numerical examples, and successfully tested by examples covering a wide range of system event types, reliability indices, number of components, and correlation properties. The method is also applied to compute the sensitivity of the first-passage probability of a building structure under stochastic excitations, modeled by use of finite elements.

1. Introduction

Sensitivity analysis is an important part of determining impacts of input variables on the function, system or performance output. Such an analysis not only provides quantitative measures that help identify relative importance of variables in terms of their impact on the results, but also facilitate the use of gradient-based optimizers in efforts to optimize the system. In risk-based decision making processes to improve or optimize a system subjected to significant uncertainties, it is essential to identify relative contributions of various input random variables in terms of parameter sensitivities of the failure probability. To this end, various sensitivity-based importance measures have been developed. Such measures quantify relative importance of random variables in terms of the difference in the failure probability caused by the changes in the distribution parameters proportional to the standard deviations or those made possible by the fixed upgrade cost [1,2].

The recent emergence of research in reliability based design optimization (RBDO) [3–8] also demands calculating parameter sensitivity of the failure probability. In fact, RBDO aims to find the values of design variables that maximize or minimize a given objective function describing the performance of the system while satisfying probabilistic constraints. A typical RBDO formulation is

$$\begin{aligned} \min_{\mathbf{d}} \quad & f_{\text{obj}}(\mathbf{d}) \\ \text{s.t.} \quad & P(E_i;\mathbf{d}) \leqslant P_i^{\text{target}},\ i = 1,\ldots,n_c \\ & \mathbf{d}^{\text{lower}} \leqslant \mathbf{d} \leqslant \mathbf{d}^{\text{upper}} \end{aligned} \tag{1}$$

where $f_{\text{obj}}(\mathbf{d})$ is the objective function of a given design optimization problem, e.g., volume, total cost and performance measure, $\mathbf{d} = \{d_1,\ldots,d_n\}$ is the set of the design variables with the lower bounds $\mathbf{d}^{\text{lower}}$ and the upper bounds $\mathbf{d}^{\text{upper}}$ (box constraints), and

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E_i and P_i^{target} respectively denote the event that the i-th constraint is violated, $i = 1, \ldots, n_c$ (or the i-th failure event), and the corresponding target failure probability. Sensitivity analysis of the probabilistic constraints with respect to design variables is a crucial part of the reliability based design optimization especially when a gradient-based optimization algorithm needs to be utilized.

In the aforementioned situations of RBDO, if the failure event is described as a system event E_{sys}, i.e., a logical function of multiple component events representing failure modes, locations or time points, parameter sensitivities of the system failure probability are needed. Among various examples of system failure events [9,10], let us consider the first passage probability of a structure subject to stochastic excitations [11–15]. This is the probability that a stochastic response $X(t)$ exceeds a given threshold x_0 at least once for a given duration $t \in (0, t_n]$. This is commonly utilized to find the probability of the failure event described within a time interval. One of the available approaches for formulating the first passage probability consists of defining the problem as a series system problem [13], i.e.,

$$P(E_{\text{sys}}) = P(x_0 < \max_{0<t\leqslant t_n} |X(t)|) = P\left(\bigcup_{i=1}^{n} |X(t_i)| > x_0\right) \tag{2}$$

where t_i is the i-th discretized time point, $i = 1,\ldots,n$. The first passage probability defined in Eq. (2) requires evaluation of the component failure probability at each time point and the statistical dependence between the failures at different time points. If a probabilistic constraint is associated with the first passage probability in RBDO [15], an efficient, reliable and robust algorithm is required to compute the system failure probability during the iterative procedure in RBDO.

In general, the sensitivity of the system failure probability with respect to a parameter θ is obtained by a chain rule, i.e.,

$$\frac{\partial P(E_{\text{sys}})}{\partial \theta} = \sum_{i=1}^{n} \frac{\partial P(E_{\text{sys}})}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial \theta} \tag{3}$$

where β_i is the reliability index of the i-th component failure event. It is noted that the impact of the correlation between component failure events on the partial derivative is assumed to be negligible. From Eq. (3), the partial derivatives of the component reliability index with respect to the design variables are available from parameter sensitivities of component reliability analysis [1,16]. However, the derivative of the system failure probability with respect to the reliability index has not yet been clearly addressed in the literature. Several methods have been developed to compute parameter sensitivities of the system failure probability. Song and Kang [10] used the Matrix-based System Reliability method [17] for computing parameter sensitivities for systems under statistical dependence, and later the method was further developed [18]. In [10] and [18], the sensitivity of the system failure probability was computed with respect to the mean and the standard deviation of the input random variables to facilitate the decision-making process and system reliability-based design optimization [7,19]. Sensitivity-based importance measures [1,2] were also computed to quantify the relative importance of the design variables. Sues and Cesare [20] proposed a method of computing parameter sensitivity of the system failure probability using the results of component reliability analysis by the first-order reliability method and Monte Carlo simulations. Song and Der Kiureghian [21] utilized the linear programing bounds method [9] in order to compute lower and upper bounds of the parameter sensitivities of general system events, even with incomplete information on component probabilities and their statistical dependence. Despite these proposed methods, computing parameter sensitivities of the system failure probability is still challenging if the system has a large number of components and/or the correlation properties of component failure events do not allow for achieving conditional independence between components given a small number of common source random variables [18].

Therefore, in this paper, a method of computing parameter sensitivity of the system failure probability is proposed using the sequential compounding method (SCM) [22] which was recently developed to compute multivariate normal integrals of general system events with a wide range of correlation properties even for those with a large number of component events. The proposed method, termed as Chun–Song–Paulino (CSP) method, is illustrated and tested by a variety of numerical examples. The CSP method is further demonstrated by application to the first passage probability of a structure described by a finite element model subjected to stochastic excitations.

The remainder of the paper is structured as follows. After a brief summary of the SCM [22], the SCM-based parameter sensitivity formulations are derived for series, parallel and general systems (cut-set system) respectively. Numerical examples test the CSP method and demonstrate its application to first-passage problems. Finally, concluding remarks and discussions on future research needs are provided.

2. Sequential compounding method

In the sequential compounding method (SCM; Kang and Song [22]), two component events coupled by a union or intersection operation in the system event are compounded sequentially until a single compound event eventually represents the system event. Each compounding procedure consists of determining the probability of the new compound event, and evaluating the correlation coefficient between the new compound event and each of the remaining component events.

First, when two events are coupled by an intersection operation, the compounding process starts by obtaining the reliability index of the compound event $E_{\text{iand}j} = E_i \cap E_j$ as

$$\beta_{\text{iand}j} = -\Phi^{-1}[P(E_i \cap E_j)] = -\Phi^{-1}[\Phi_2(-\beta_i, -\beta_j; \rho_{i,j})] \tag{4}$$

where $\beta_{\text{iand}j}$ denotes the reliability index of the compound event, $\Phi(\cdot)$ is the marginal cumulative distribution function (CDF) of the standard normal distribution, and $\Phi_2(\cdot)$ is the joint CDF of the bi-variate standard normal distribution. $\rho_{i,j}$ is the correlation coefficient between the standard normal random variables representing E_i and E_j, which could be obtained from the inner-product of the negative normalized vectors of the design points [23]. After $\beta_{\text{iand}j}$ is obtained, the correlation coefficient between $E_{\text{iand}j}$ and each of the remaining component events $E_k, k = 1, \ldots, n, k \neq i, j$, denoted by $\rho_{(\text{iand}j),k}$, is computed. The correlation coefficient is determined such that the compound event can represent $E_i \cap E_j$ accurately in computing the probability of $E_i \cap E_j \cap E_k$, i.e.,

$$\Phi_3(-\beta_i, -\beta_j, -\beta_k; \rho_{i,j}, \rho_{i,k}, \rho_{j,k}) = \Phi_2(-\beta_{\text{iand}j}, -\beta_k; \rho_{(\text{iand}j),k}) \tag{5}$$

In Eq. (5), $\rho_{(\text{iand}j),k}$ is the only unknown variable, which can be obtained numerically by nonlinear programing

$$\begin{aligned} \min_{\rho_{(\text{iand}j),k}} \quad & \left|\Phi_3(-\beta_i, -\beta_j, -\beta_k; \rho_{i,j}, \rho_{i,k}, \rho_{j,k}) - \Phi_2(-\beta_{\text{iand}j}, -\beta_k; \rho_{(\text{iand}j),k})\right| \\ \text{s.t.} \quad & -1 \leqslant \rho_{(\text{iand}j),k} \leqslant 1 \end{aligned} \tag{6}$$

The multi-fold integrals of the joint CDFs in the optimization problem can be performed by efficient algorithms such as the one by Genz [24]. To further reduce the computational costs for solving Eq. (5) during the optimization process, Kang and Song [22] proposed an approximate procedure as well, which deals with single-fold integrals only.