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Reliability assessment of reinforced concrete columns based on the P–M interaction diagram using AFOSM

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ABSTRACT

This paper presents a new approach for evaluating the reliability index and most probable failure point (MPFP) of a reinforced concrete (RC) column using the advanced first-order second-moment reliability method without Monte-Carlo simulations. The P–M interaction diagram (PMID) is selected as the limit state function of an RC column. The strength parameters of an RC column include the material properties and geometric properties of the cross section of an RC column. The strength and load parameters are considered as random variables. An iterative solution scheme with double iteration loops is adopted for obtaining the MPFP and reliability index. The continuous and differentiable PMID is constructed with discretely defined sampling points of the PMID using the cubic spline interpolation. The sensitivities of the PMID are calculated through the direct differentiation of the cubic spline and sampling points of the PMID. Detailed expressions of the sensitivities of the PMID with respect to the random variables are presented. The validity of the proposed method is demonstrated through a simple rectangular column and the pylon of a real cable-stayed bridge. It is shown that the proposed method yields physically meaningful solutions efficiently for the examples presented.

1. Introduction

Various types of reinforced concrete (RC) columns ranging from columns of buildings to pylons of cable-supported bridges are designed based on the P–M interaction diagrams (PMID) [1], which define the limit states of columns subject to combined axial and bending actions. Since the failure of a column may result in the total collapse of a structure, the precise estimation of the failure point of a column is one of the most important issues in the design and the reliability assessment of a column, especially in code calibrations.

Various approaches [2–8] have been proposed to evaluate reliability indices of RC columns since the statistical characteristics of the strengths of RC columns were evaluated through the Monte-Carlo simulations by Ellingwood [9] and Grant et al. [10]. Stewart and Attard [2] and Szerszen et al. [3] assumed the eccentricity of total load effect, which is the ratio of bending moment to axial force, to be a deterministic variable for the reliability assessment of RC columns. The uncertainty of the eccentricity was taken into account in the works by Hong and Zhou [4] and Jiang and Yang [5]. Mirza [6] estimated the moment capacities of RC columns for a fixed axial force, while Frangopol et al. [7] and Milner et al. [8] performed the reliability analyses for load paths determined by load correlations. The statistical characteristics of the strength of an RC column are obtained through the Monte-Carlo simulation [2,3,6–10].

The aforementioned studies are based on one common assumption that the strength of an RC column can be pre-determined on the PMID by a load condition. That is, the strength of an RC column can be defined as an intersection point in the P–M space between the PMID and a straight line connecting the origin and total load effect [2–5] or between the PMID and pre-defined load path [6–8]. With this assumption, the limit state function of an RC column is simply expressed as the assumed strength minus the total load effect applied to the column, which is an approximation of a real limit state function but a convenient form to apply a traditional reliability analysis scheme. However, the approximated limit state function may lead to erroneous results because the real strength of an RC column at failure depends on not only the total load effect but also the statistical characteristics of all random variables. The PMID itself defines the failure and safe states of an RC column, and thus the PMID of an RC column should be adopted as the limit

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state function for accurate and reliable assessment. Another shortcoming of the previous studies is that statistical variations are applied directly to internal forces [2–8]. Since the internal forces simply represent the load effects induced by the external load components, the load components should be chosen as independent random variables rather than internal forces, and thus the statistical variations should be taken into account in the individual load component.

This paper presents a new approach to estimate the MPFP and reliability index of a short RC column, in which the nonlinear P-delta effect can be neglected, based on the advanced first-order second-moment reliability method (AFOSM) [11]. The PMID of an RC column is adopted as the limit state function for the AFOSM without employing any assumption on the strength of an RC column, and external load components rather than internal forces are selected as independent random variables. The PMID representing the column strength depends upon the material and geometric properties of cross section of a column, which are also considered as random variables. The material properties include the compressive strength of concrete, the Young's modulus and the yield strength of each reinforcing bar. Meanwhile, the gross area of a cross section, the areas and the locations of reinforcing bars are the random variables on the geometric properties. The proposed method does not require Monte-Carlo simulations to obtain the statistical properties of the column strength. The reliability indices are directly calculated in the AFOSM using the sensitivities of the PMID with respect to random variables. Since the PMID is generally nonlinear with respect to the random variables, the Hasofer–Lind–Rackwitz–Fiessler (HL–RF) algorithm with the gradient projection method [12] is adopted to solve the minimization problem that defines the MPFP in the AFOSM.

The HL–RF algorithm requires the first-order sensitivities of the PMID with respect to the random variables. However, the PMID is defined at discrete sampling points corresponding to given locations of the neutral axis of a cross section. Therefore, a continuous and differentiable PMID should be constructed with the discrete sampling points to calculate the sensitivities of the PMID. The cubic spline interpolation [13], which is the collection of piecewise cubic polynomials interpolating two adjacent sampling points of the PMID, is employed. The coefficients of the cubic spline are determined based on the continuity requirements up to the second-order derivatives at the boundaries between two adjacent segments of the cubic spline. The direct differentiation of each segment of the cubic spline with respect to the random variables yields the sensitivities of the PMID required in the AFOSM.

The validity of the proposed method is demonstrated through two examples: (1) a simple rectangular section found frequently in a building structure; and (2) the pylon section of a cable-stayed bridge in Korea. Detailed investigations on the variations of the MPFP with different load combinations are presented. It is shown that a dominant load component governs the limit state of columns, and the variations of the material and geometric properties have a rather minor effect on the failure of columns.

2. Formulation of the AFOSM for PMID

The PMID of an RC column subject to combined axial and flexural load is implicitly defined in the axial force (P)-moment (M) space as follows:

$$\Phi = \Phi(\mathbf{F}, \mathbf{A}) = 0 \tag{1}$$

where $\mathbf{F} = (P, M)^T$, and $\mathbf{A}$ is the curve parameter vector of the PMID. The curve parameters are determined based on the strength parameters representing the material and geometric properties of a cross section. The material properties include the compressive strength of concrete, f_{ck}, the Young's modulus of the reinforcing bar, E_s, and the yield strength of the reinforcing bar, f_y. The definitions of the geometric properties for a typical cross section of an RC column are illustrated in Fig. 1(a). The geometric properties consist of the gross area of a cross section as well as the area and position of each reinforcing bar. The strength parameters of an RC column are conveniently written in one vector.

$$\mathbf{s} = (s_j) = (f_{ck}, f_y, E_s, A_{gt}, A_{s,1}, \ldots, A_{s,k}, \ldots, A_{s,m}, y_{s,1}, \ldots, y_{s,k}, \ldots, y_{s,m})^T \tag{2}$$

where $m, A_{gt}, A_{s,k}, y_{s,k}$ are the number of reinforcing bars, the gross area of a cross section, the area and position of the k-th reinforcing bar, respectively. The position of a reinforcing bar is measured from the extreme compression fiber of the cross section to the center of the reinforcing bar as shown in Fig. 1(a). All random variables in Eq. (2) are assumed to be statistically independent to each other.

The PMID given in Eq. (1) defines the limit state of an RC column. That is, $\Phi(\mathbf{F}_q, \mathbf{A}) > 0$ and $\Phi(\mathbf{F}_q, \mathbf{A}) < 0$ represent the safe and failure states of the RC column, respectively, and therefore $\Phi(\mathbf{F}_q, \mathbf{A}) = 0$ depicts the limit state of the RC column. Here, $\mathbf{F}_q$ is the internal force vector representing the load effects of external load components such as dead load, live load, wind load, etc. Although each external load component may have nonlinear load effects on an RC column, linear relations between the internal forces and the external loads are assumed:

$$\mathbf{F}_q = \begin{pmatrix} P_q \\ M_q \end{pmatrix} = \mathbf{Cq} \tag{3}$$

Here, $\mathbf{C}$ and $\mathbf{q}$ are the load effect matrix and load parameter vector, respectively. Each column of the load effect matrix is composed of the load effects calculated in the structural analysis for the nominal value of the corresponding load component. The load parameter vector represents the statistical properties of the load components. Each load parameter has the nominal value of 1, and its mean and coefficient of variation (COV) become the bias factor and the COV of the original load component, respectively. The statistical distributions of the load parameters follow those of the original load components.

The strength parameters of an RC column and the load parameters are considered to be random variables in this study. For the compactness of forthcoming derivations, the random variables are written in one vector, $\mathbf{X} = (\mathbf{q}, \mathbf{s})^T$. In case that all random variables are normally distributed and statistically independent to each other, the MPFP of an RC column is estimated using the AFOSM, in which the MPFP is defined as the solution of the following minimization problem [11]:

$$\underset{\bar{\mathbf{X}}}{\text{Min}}\, \beta^2 = \|\bar{\mathbf{X}}\|_2^2 \quad \text{subject to} \quad \bar{\Phi}(\bar{\mathbf{X}}) = 0 \tag{4}$$

where β and $\|\cdot\|_2$ denote the reliability index and the 2-norm of a vector, respectively, while the overbarred variables indicate standardized random variables and $\bar{\Phi}(\bar{\mathbf{X}}) = \Phi(\mathbf{X})$. The minimization problem given in Eq. (4) is solved iteratively by the Hasofer–Lind–Rackwitz–Fiessler (HL–RF) algorithm with the gradient projection method [12]. The equivalent normal distributions estimated by the Rackwitz–Fiessler method [14] are utilized for nonnormal random variables.

The sensitivity of the PMID with respect to the random variables is required to solve the minimization problem given in Eq. (4), and is calculated by the direct differentiation of the PMID using the chain-rule.

$$\begin{pmatrix} \dfrac{\partial \bar{\Phi}}{\partial \bar{\mathbf{q}}} \\ \dfrac{\partial \bar{\Phi}}{\partial \bar{\mathbf{s}}} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial \bar{\Phi}}{\partial \mathbf{F}} \dfrac{\partial \mathbf{F}}{\partial \bar{\mathbf{q}}} + \dfrac{\partial \bar{\Phi}}{\partial \mathbf{A}} \dfrac{\partial \mathbf{A}}{\partial \bar{\mathbf{q}}} \\ \dfrac{\partial \bar{\Phi}}{\partial \mathbf{F}} \dfrac{\partial \mathbf{F}}{\partial \bar{\mathbf{s}}} + \dfrac{\partial \bar{\Phi}}{\partial \mathbf{A}} \dfrac{\partial \mathbf{A}}{\partial \bar{\mathbf{s}}} \end{pmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{F}}{\partial \bar{\mathbf{q}}} & \dfrac{\partial \mathbf{A}}{\partial \bar{\mathbf{q}}} \\ \dfrac{\partial \mathbf{F}}{\partial \bar{\mathbf{s}}} & \dfrac{\partial \mathbf{A}}{\partial \bar{\mathbf{s}}} \end{bmatrix} \begin{pmatrix} \dfrac{\partial \bar{\Phi}}{\partial \mathbf{F}} \\ \dfrac{\partial \bar{\Phi}}{\partial \mathbf{A}} \end{pmatrix} = \mathbf{Q} \begin{pmatrix} \dfrac{\partial \Phi}{\partial \mathbf{F}} \\ \dfrac{\partial \Phi}{\partial \mathbf{A}} \end{pmatrix} \tag{5}$$