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Analytical structural reliability analysis of a suspended cable

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ABSTRACT

Suspended cables, including transmission lines, suspension bridge cables, and edge cables of roof structures, feature in high profile and large span projects for architectural reasons, for their functional efficiency, and for ease of construction, particularly over large spans. A suspended cable predominantly reacts external loads by means of axial tension, and is, therefore, able to make full use of the material strength. Because of the slenderness of the suspended cable, the structural response is nonlinear, even if the material property is within the elastic range. From a mechanics perspective, therefore, these types of structure exhibit high levels of geometric non-linearity. For this reason, the nonlinear relationships between tension force, normal displacement, and the external loads need to be considered. In aiming to determine the structural safety of a suspended cable, and to understand which uncertainty features have the greatest influence, these relationships are written within a probabilistic framework.

This article briefly sets the analysis of suspended cables within the context of geometrically nonlinear elastic structures and corresponding finite element analysis methodologies. Analytical solutions to the tension and normal displacement of a suspended cable subjected to external loading are presented. Nonlinear performance functions, in the form of either the cable tension or normal displacement are stated. Analytical expressions for the required gradients of the performance function of a suspended cable with respect to the basic variables under static loads are developed. The structural reliability of a suspended cable is studied using a first-order reliability method (FOSM) and verified by comparison with Monte Carlo simulation (MCS) and Monte Carlo simulation based optimization principles (MCOP) for a number of examples. Load cases including, wind, snow, and temperature variation are included.

1. Introduction

Suspended cables, including transmission lines, suspension bridges and roofs, are widely used as long-span engineering solutions. Unlike conventional structures, a suspended cable is a flexible structural system, the shape of which cannot be prescribed, but must take a 'form-found' configuration determined by equilibrium and geometric constraints on the basis of a predefined initial stress state [1–3]. Because of the slenderness and flexibility of a suspended cable, the structural responses are nonlinear even if the material property is within the elastic range. For this reason, geometric nonlinearity should be considered in the analysis of a suspended cable. The deterministic geometrically nonlinear analysis of a suspended cable is explained in references [4–6] and others. In reality, the variables affecting the safety of structure are random because these parameters contain uncertainties introduced in the form of epistemic uncertainties in the design process, and aleatoric uncertainties in the form of material characteristics, construction tolerances, and service conditions including loading. An accurate prediction of the performance of an analytically described suspended cable in the presence of uncertainty is presented in this paper.

Previously published work on the reliability analysis of cable structures has mainly focused on evaluating the reliability of geometrically nonlinear structures through the use of the finite element method. For instance, Liu and Kiureghian [7] introduced the finite element-based reliability method, formulated using FOSM and SORM principles, for geometrically nonlinear elastic structures, and created a general purpose reliability analysis code to evaluate structural reliabilities. Xinpei Zhang [8], proposed an algorithm to calculate the safety index with limit states based on element strength and in service performance of a cable structure using a nodal displacement algorithm and the checking point (JC) method of reliability. Imai and Frangopol [9,10] and Frangopol and Imai [11] considered the nonlinear relationships between strains and displacements and investigated the system reliability

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evaluation of suspension bridges by a probabilistic finite element analysis approach. The above methods, combining probabilistic theory with the traditional finite element analysis, are one of the more practically effective methods to analyse the reliability of complex structures in the broadest sense. However, little work has been done on the reliability analysis of a suspended cable based on classical analytical solutions at present, which, once available, negates the need to consider further numerical modelling, and offers both computational efficiency and clear definitions of assumptions that may contribute to otherwise unknown or unclear epistemic uncertainty.

The aim of this paper is to formulate and quantify the reliability of a suspended cable on the basis of a classical analytical structural mechanics solution. In the following section, the theory of suspended cable is described and the analytical solution for the tension of a suspended cable is provided. In Section 3, the limit state function of suspended cable is established and the gradients of the limit state function with respect to the basic random variables are deduced. In Section 4, the computational accuracy of FOSM used in suspended cable is demonstrated by comparison with Monte Carlo simulation (MCS) and Monte Carlo simulation based optimization principle (MCOP) by using an example of transmission line (suspended cable). Conclusions from the present study are drawn in the last section.

2. Analytic solution of a suspended cable

2.1. Selection of cable equation

Following the theoretical basis of Irvine [1], we consider here a profile adopted by a uniform cable suspended between two rigid supports that are at the same level and subjected to a uniform distributed load mg along cable length, as shown in Fig. 1. It is assumed that the cable: (1) is perfectly flexible and devoid of flexural rigidity; (2) can sustain only tensile forces; (2) is composed of a homogeneous material which is linearly elastic.

Considering the sketches in Fig. 1, the vertical and horizontal equilibrium of the isolated element of the cable located at (x,z) require that:

$$\mathrm{d}V + mg\mathrm{d}s = 0 \tag{1}$$

$$\mathrm{d}H = 0, \tag{2}$$

where, H is the horizontal component of cable tension. H is constant everywhere f no longitudinal loads act on the cable, or Eq. (2) may be directly integrated; V is the vertical component of cable tension and can be written as:

$$V = H\tan\theta = H\frac{\mathrm{d}z}{\mathrm{d}x}. \tag{3}$$

Differentiating Eq. (3) with respect to x and substituting it into Eq. (1), and noting that the following geometric constraint that must be satisfied:

$$\left(\frac{\mathrm{d}x}{\mathrm{d}s}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}s}\right)^2 = 1, \tag{4}$$

the governing differential equation of the cable is obtained as:

$$H\frac{\mathrm{d}^2z}{\mathrm{d}x^2} + mg\sqrt{1 + \left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2} = 0. \tag{5}$$

Solving the differential Eq. (5), the profile function of the cable that satisfies the boundary conditions in Fig. 1 can be obtained as:

$$z = \frac{H}{q}\left[\cosh\frac{mgl}{2H} - \cosh\left(\frac{mgx}{H} - \frac{mgl}{2H}\right)\right] \tag{6}$$

This is a catenary equation of a suspended cable, fully determined by the coordinate x at any point, (for example, the sag f at mid-span). If the sag at mid-span of cable is f, namely, $x = l/2$, $z = f$, the horizontal component of cable tension H can be calculated from Eq. (6), as in:

$$f = \frac{H}{mg}\left(\cosh\frac{mgl}{2H} - 1\right). \tag{7}$$

Because the catenary equation of the suspended cable and the equations derived from it all involve hyperbolic functions and, therefore, transcendental equations as functions of the problem defining variables, the solution of the system of differential equations is overly complicated. It is, however, possible to derive some relatively simple solutions for specific loading and boundary conditions, such as for a profile under a distributed load mg along the cable span, for example. In the context of a transmission line idealisation, we may consider the profile of a uniform cable spanning between two supports at the same level, generated by a uniformly distributed vertical load mg, as shown in Fig. 2. The horizontal equilibrium of the isolated element of cable is the same as defined in Eq. (2). The vertical equilibrium of an element requires that:

$$\mathrm{d}V + mg\mathrm{d}s = 0 \tag{8}$$

Differentiating Eq. (3) with respect to x and substituting it into Eq. (8), then:

$$H\frac{\mathrm{d}^2z}{\mathrm{d}x^2} = -mg \tag{9}$$

If the profile is relatively flat, so that the ratio of sag span is 1:8 or less, the differential equation governing the vertical equilibrium of an element is accurately specified by Eq. (9) [1]. Integrating Eq.

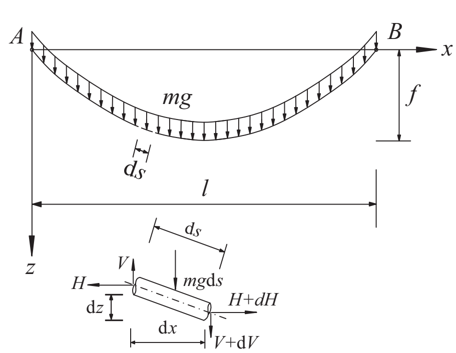

Fig. 1. A cable under a distributed load along the cable length and equilibrium of an element of cable.

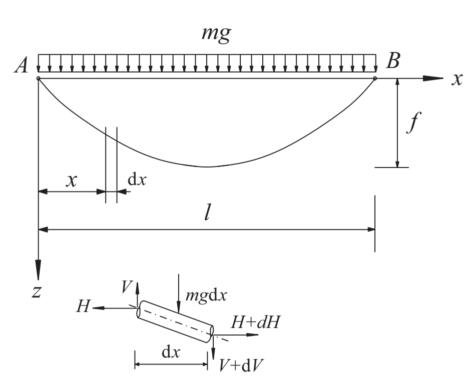

Fig. 2. A cable under a distributed load and equilibrium of an element of the cable.