Structural Safety 58 (2016) 52–59

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

An algorithm for finding a sequence of design points in reliability analysis

Ziqi Wang [a,*], Marco Broccardo [b], Armen Der Kiureghian [b]

[a] Department of Bridge Engineering, Southwest Jiaotong University, Chengdu, China
[b] Department of Civil and Environmental Engineering, University of California, Berkeley, CA, USA

ARTICLE INFO

Article history:
Received 2 March 2015
Received in revised form 31 July 2015
Accepted 17 September 2015
Available online 1 October 2015

Keywords:
Design point
Nonlinear analysis
Optimization algorithm
Quasi-Newton
Reliability analysis

ABSTRACT

In the analysis of structural reliability, often a sequence of design points associated with a set of thresholds are sought in order to determine the tail distribution of a response quantity. In this paper, after a brief review of methods for determining the design point, an inverse reliability method named the λ-method is introduced for efficiently determining the sequence of design points. The λ-method uses Broyden's "good" method to solve a set of nonlinear simultaneous equations to find the design points for the values of an implicitly defined threshold that is associated with parameter λ. In a special parameter setting, the λ parameter equals the reliability index, thus allowing convenient implementation of the method. Three numerical examples illustrate the accuracy and efficiency of the proposed method.
© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

In analysis of structural reliability, there is often interest in solving the reliability problem for a range of threshold values. Three specific examples include: (a) determining the probability distribution of a structural response quantity by, e.g., probabilistic finite-element analysis; (b) determining the fragility (conditional probability of failure) of a structure for a given range of demand thresholds; and (c) in stochastic dynamic analysis for determining such statistics of the response, as up-crossing rates of various thresholds or probabilities of exceeding a range of thresholds. In general, these problems can be formulated in terms of a limit-state function of the form $g(\mathbf{x}, r) = r - R(\mathbf{x})$, where $\mathbf{x}$ denotes a vector of random variables representing uncertain structural or load values, $R(\mathbf{x})$ denotes the response or capacity quantity of interest, and r is the threshold. The objective is to solve the reliability problem for a range of values of r. For the specific examples above, the corresponding probability expressions are as follows: (a) the cumulative distribution function (CDF) of response $R(\mathbf{x})$ is given by $F_R(r) = \Pr[R(\mathbf{x}) \leqslant r] = 1 - \Pr[g(\mathbf{x}, r) \leqslant 0]$; (b) the fragility function for capacity $R(\mathbf{x})$ is given by $\Pr[R(\mathbf{x}) \leqslant D | D = r] = 1 - \Pr[g(\mathbf{x}, r) \leqslant 0]$, where r now denotes the threshold of a demand D; and (c) for a nonlinear stochastic response process $R(\mathbf{x}, t)$, the first-order solution of the problem $\Pr[r \leqslant R(\mathbf{x}, t)] = \Pr[g(\mathbf{x}, t, r) \leqslant 0]$ leads to a tail-equivalent linear system for which all statistics of interest for threshold r can be determined by linear random vibration analysis (see Fujimura and Der Kiureghian [16] and Section 7 in this paper). For the sake of convenience, hereafter we call the above class of problems *threshold-reliability problems*.

Since analytical solutions of reliability problems are often unavailable, approximate techniques, such as the first- and second-order reliability methods (FORM and SORM) [1,2], response surface methods (RSM) [3,4], various sampling techniques [5–7], and expansion methods [8] are used. Several popular methods among these, including FORM, SORM and importance sampling (IS) [5], require determination of the so-called "design point" for each limit-state function. This is the point on the limit-state surface in a transformed standard normal space, which has minimum distance from the origin [1]. This point has the property of having the highest probability density among all failure points in the standard normal space. Hence, it is an optimal point for constructing approximations of the surface (first-order in FORM, second-order in SORM) or as a center of sampling in IS. Furthermore, in FORM, the distance from the origin to the design point, known as the *reliability index*, is directly related to the first-order approximation of the failure probability. The design point is usually obtained by solving a constrained optimization problem by a gradient-based algorithm (see, e.g., [9–13,20]). When the reliability problem is to be solved for a range of thresholds, a sequence of design points must be computed. This can be a costly computation when evaluation of the limit-state function or its gradient involves extensive numerical calculations.

* Corresponding author.

http://dx.doi.org/10.1016/j.strusafe.2015.09.004
0167-4730/© 2015 Elsevier Ltd. All rights reserved.

In this paper, we present a new method for determining the sequence of design points for the class of threshold-reliability problems. Named the λ-*method*, the proposed method finds the sequence of design points through inverse reliability analysis. That is, instead of finding the reliability index for a sequence of thresholds, it finds the threshold values for a sequence of prescribed values of the parameter λ, which in a special case is identical to the reliability index. Similar to existing methods, the λ-method employs the gradient of the limit-state function; however, it is far more efficient than existing methods for "forward" reliability analysis for a sequence of thresholds. It is important to note that the λ-method has only one convergence criterion, while standard algorithms for finding the design point have two criteria: The point must be on the limit-state surface, and it must be an origin projection point [13]. This is one of the reasons the proposed method is more efficient.

After a brief review of the conventional formulation for finding the design point, we present the λ-method and develop a specific algorithm for its solution. We end the paper with three numerical examples that demonstrate the superior computational efficiency of the proposed method relative to the most widely used existing algorithm for finding the sequence of design points.

2. Existing algorithm for finding a sequence of design points

As mentioned earlier, the design point in reliability analysis is found in a transformed space of standard normal random variables. Let $\mathbf{u} = \mathbf{T}(\mathbf{x})$ be a one-to-one mapping from the space of original random variables $\mathbf{x}$ to the space of standard normal random variables $\mathbf{u} = \mathrm{N}(0, \mathbf{I})$, where $\mathrm{N}(\mathbf{M}, \Sigma)$ denotes the joint normal distribution with mean vector $\mathbf{M}$ and covariance matrix Σ. Conditions under which such a transformation exists and its forms for different classes of the joint distribution of $\mathbf{x}$ can be found in Der Kiureghian [2]. Let $G(\mathbf{u}, r) = r - R(\mathbf{T}^{-1}(\mathbf{u}))$ denote the limit-state function in the standard normal space for a threshold-reliability problem. The design point is the solution to the constrained optimization problem

$$\mathbf{u}^* = \arg\min\{\|\mathbf{u}\| \,|\, G(\mathbf{u}, r) = 0\} \tag{1}$$

Most efficient algorithms for solving this problem employ the gradient vector. In the reliability community, the HLRF algorithm and its modified versions are most widely used. Originally introduced by Hasofer and Lind [9] in the context of second-moment reliability analysis, the algorithm has been generalized by Rackwitz and Fiessler [10] to account for distribution information and modified by Liu and Der Kiureghian [11] and Zhang and Der Kiureghian [12] to control the step size and improve its convergence. Other gradient-based algorithms for solving this problem include the Abdo–Rackwitz algorithm [14], the Polak–He algorithm [15] and other general-purpose optimization algorithms, such as gradient projection method and sequential quadratic programming. A comparative study of several algorithms used to solve this problem is reported in Liu and Der Kiureghian [11].

A gradient-based algorithm naturally requires the existence of the gradient row-vector $\nabla_{\mathbf{u}} G(\mathbf{u}, r) = [\partial G/\partial u_1 \cdots \partial G/\partial u_n]$, where n denotes the number of random variables. The algorithm typically constructs a sequence of trial points according to the rule

$$\mathbf{u}_{i+1} = \mathbf{u}_i + s_i \mathbf{d}_i, \quad i = 0, 1, \ldots \tag{2}$$

where $\mathbf{u}_0$ is the initial trial point, $\mathbf{d}_i$ is a search direction that involves the gradient vector, and s_i is the step size for the i th iteration. Algorithms are different in their choice of the search direction and step size. The sequence is considered converged when the following equalities are satisfied within acceptable tolerances:

$$G(\mathbf{u}, r) = 0 \tag{3}$$

$$\mathbf{u} + \frac{\nabla_{\mathbf{u}} G(\mathbf{u}, r)^T}{\|\nabla_{\mathbf{u}} G(\mathbf{u}, r)\|} \|\mathbf{u}\| = \mathbf{0} \tag{4}$$

Eq. (3) ensures that the design point is located on the limit-state surface. Eq. (4) assures that the design point is an origin-project point, i.e., that the gradient vector at the solution point is directed towards the origin of the standard normal space. This equation also implies that the gradient and the design point vector have opposite directions. Once the design point is found, the reliability index is computed as

$$\beta = -\frac{\nabla_{\mathbf{u}} G(\mathbf{u}^*, r)^{\mathrm{T}}}{\|\nabla_{\mathbf{u}} G(\mathbf{u}^*, r)\|} \mathbf{u}^* \tag{5}$$

When the design points for a sequence of thresholds $r_1 < r_2 < \cdots < r_p$ are of interest, the above problem must be solved repeatedly. However, as shown in Fujimura and Der Kiureghian [16], advantage can be gained by using a projection method to obtain a good starting point for each iteration. Having obtained $\mathbf{u}^*_{i-1} = \mathbf{u}^*(r_{i-1})$ and $\mathbf{u}^*_i = \mathbf{u}^*(r_i)$, a good approximation of $\mathbf{u}^*_{i+1}$ is

$$\hat{\mathbf{u}}^*_{i+1} = \mathbf{u}^*_i + \theta \frac{\mathbf{u}^*_i - \mathbf{u}^*_{i-1}}{\|\mathbf{u}^*_i - \mathbf{u}^*_{i-1}\|} \tag{6}$$

where θ is obtained by solving $G(\hat{\mathbf{u}}^*_{i+1}, r_{i+1}) = 0$. $\hat{\mathbf{u}}^*_{i+1}$ may now be used as a starting point for finding $\mathbf{u}^*_{i+1}$. Fujimura and Der Kiureghian [16] have shown that significant savings in computation can be achieved by employing this formulation.

3. Formulation of the λ-method

For the threshold-reliability problem, the gradient vector has the form $\nabla_{\mathbf{u}} G(\mathbf{u}, r) = -\nabla_{\mathbf{u}} R(\mathbf{u})$, where $\nabla_{\mathbf{u}} R(\mathbf{u}) = \nabla_{\mathbf{x}} R(\mathbf{x}) \mathbf{J}_{\mathbf{x},\mathbf{u}}$ in which $\nabla_{\mathbf{x}} R(\mathbf{x})$ is the gradient vector of the response or capacity function and $\mathbf{J}_{\mathbf{x},\mathbf{u}}$ denotes the Jacobian matrix of the inverse transformation $\mathbf{x} = \mathbf{T}^{-1}(\mathbf{u})$. Using the above relation in (4), the second convergence criterion takes the form

$$\mathbf{u} - \frac{\nabla_{\mathbf{u}} R(\mathbf{u})^{\mathrm{T}}}{\|\nabla_{\mathbf{u}} R(\mathbf{u})\|} \|\mathbf{u}\| = 0 \tag{7}$$

We modify this form to read

$$\mathbf{u} - \frac{\lambda}{\|\nabla_{\mathbf{u}} R(\mathbf{u})\|^m} \nabla_{\mathbf{u}} R(\mathbf{u})^{\mathrm{T}} = 0 \tag{8}$$

where λ and m are parameters.

Suppose, for a given λ and m, $\mathbf{u}^*$ is a solution of (8). It follows that the vector $\nabla_{\mathbf{u}} R(\mathbf{u}^*)$ and, therefore, the gradient vector of the limit-state function at $\mathbf{u}^*$ are collinear with $\mathbf{u}^*$, i.e., the gradient vector passes through the origin. It follows that there is a threshold r^* for which $\mathbf{u}^*$ is the design point. This threshold is obtained by setting $G(\mathbf{u}^*, r^*) = 0$, which yields $r^* = R(\mathbf{u}^*)$. This is essentially an inverse-reliability analysis method. By solving (8) for a set of λ values and computing the corresponding thresholds, we obtain a sequence of thresholds and corresponding design points, which can then be used for reliability analysis by FORM, SORM or IS. In particular, the reliability indices are computed according to

$$\beta = \frac{\nabla_{\mathbf{u}} R(\mathbf{u}^*)^{\mathrm{T}}}{\|\nabla_{\mathbf{u}} R(\mathbf{u}^*)\|} \mathbf{u}^* \tag{9}$$

Though (8) involves the gradient vector of the response, we treat it as a set of nonlinear equations rather than differential equations. This is because the equation does not involve the response $R(\mathbf{u})$. Furthermore, it is important to observe that (8) does not involve the threshold r. This is a result of the particular form of the limit-state function that we have defined for the threshold-reliability problem.

Download English Version:

https://daneshyari.com/en/article/307487

Download Persian Version:

https://daneshyari.com/article/307487

Daneshyari.com