Structural Safety 58 (2016) 69–78

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

An efficient framework for the elasto-plastic reliability assessment of uncertain wind excited systems

Pietro Tabbuso [a], Seymour M.J. Spence [b,*], Luigi Palizzolo [a], Antonina Pirrotta [a], Ahsan Kareem [c]

[a] Department of Civil, Environmental, Aerospace, Materials Engineering (DICAM), University of Palermo, Viale delle Scienze, 90128 Palermo, Italy
[b] Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109, USA
[c] NatHaz Modeling Laboratory, Department of Civil and Environmental Engineering and Earth Sciences, University of Notre Dame, Notre Dame, IN 46556, USA

ARTICLE INFO

Article history:
Received 11 June 2015
Received in revised form 2 September 2015
Accepted 7 September 2015
Available online 2 October 2015

Keywords:
Elasto-plastic structures
Dynamic shakedown
Wind loads
Dynamic wind effects
Reliability analysis
Subset Simulation

ABSTRACT

In this paper a method to efficiently evaluate the reliability of elastic-perfectly plastic structures is proposed. The method is based on combining dynamic shakedown theory with Subset Simulation. In particular, focus is on describing the shakedown behavior of uncertain elasto-plastic systems driven by stochastic wind loads. The ability of the structure to shakedown is assumed as a limit state separating plastic collapse from a safe, if not elastic, state of the structure. The limit state is therefore evaluated in terms of a probabilistic load multiplier estimated through solving a series of linear programming problems posed in terms of the responses of the underlying linear elastic model and self-stress distribution. The efficiency of the proposed procedure is guaranteed by the simplicity of the mathematical programming problem, the underlying structural model solved at each iteration, and the efficiency of Subset Simulation. The rigor of the approach is assured by the dynamic shakedown theory. The applicability of the framework is illustrated on a steel frame example.

1. Introduction

The unavoidably aleatory and uncertain nature of the environment in which building systems are constructed, as well as the inevitable epistemic and knowledge uncertainties involved in describing such an environment, implies the necessity of using probabilistic approaches for assessing the performance of structural systems. This realization was the driving force behind the development of reliability-based approaches in civil engineering [1,2], and the subsequent development of design codes based on reliability theory [3,4]. It is also the basis on which state-of-the-art performance-based design is founded [5,6]. Recently there has been significant interest in developing specific reliability-based procedures for assessing the performance of wind excited structures [7–17]. However, as in classic reliability analysis, the limit states indicating structural failure are associated with first yield of the structural system. The situation is somewhat different in seismic engineering due to the importance of the post-yield behavior in defining adequate performance. For this reason, specialized methods, such as incremental dynamic analysis (IDA), have been developed with the aim of bringing together probabilistic design principles and step-by-step non-linear analysis [18]. In the design of wind excited structures, on the other hand, engineers generally do not explore the behavior of the structural system beyond the elastic limit. Probably the main reason for this can be found in societys intolerance towards damage of buildings due to wind storms. The downside of this is that the structural systems of many buildings are designed with no knowledge of their inelastic behavior, potentially leaving them exposed to undesirable collapse scenarios or at least unknown post-yield behavior. From a research perspective, the problem of understanding and modeling the inelastic behavior of wind excited structures has been the subject of a number of works [19–26] including the application of pushover analysis [27]. The main difficulty in defining a modeling procedure that can account for this effect is the extremely long duration of wind storms. Indeed, this characteristic practically eliminates the possibility of using methods such as IDA as they require non-linear dynamic integration of the entire load history, which constitutes a computationally daunting task [28,29]. An alternative approach for engineered building systems is presented by the well-established plastic theorem approaches [30–32]. Indeed, recent computational advances make these methods an attractive alternative for rapidly assessing the general behavior of ductile structures such as steel and concrete frames [33–35]. These methods also provide a complete picture of the post-yield behavior

* Corresponding author. Tel.: +1 734 764 8419; fax: +1 734 764 4292.
E-mail addresses: pietro.tabbuso@unipa.it (P. Tabbuso), smjs@umich.edu (S.M.J. Spence), luigi.palizzolo@unipa.it (L. Palizzolo), antonina.pirrotta@unipa.it (A. Pirrotta), kareem@nd.edu (A. Kareem).

http://dx.doi.org/10.1016/j.strusafe.2015.09.001
0167-4730/

of the system, indicating for example when the system is in shakedown or low cycle fatigue (plastic shakedown) which can be of importance for structures subject to long duration cyclic loads such as extreme wind excitation.

Shakedown analysis is classically carried out under a quasi-static loading scenario. Under these conditions, the goal is to understand whether an elasto-plastic structure subject to loads varying within a specified domain will eventually respond in a purely elastic manner after a finite amount of plastic deformation and is based on the well-known Bleich-Melan and Koiter theorems. In the last decades, many applications have been treated with this approach [35–38], including recent applications where both the loads and the strength parameters have been considered as uncertain [39,40]. When dynamic effects are important, dynamic shakedown analysis becomes necessary. This concept was first introduced by Ceradini [41] through the development of a lower bound theorem which took the form of conditions under which an elasto-plastic solid subject to an infinite dynamic load history will shakedown. The main difference between quasi-static and dynamic shakedown analysis is that in the first the loading history is defined by a convex domain in which the loads may be repeated indefinitely, while in the second the load history must be fully specified. In addition, unlike the quasi-static theory, in dynamic shakedown the solution will in general depend on the initial conditions of the fictitious elastic response. This situation generally makes the application of Ceradini's theorem considerably more computationally involved compared to the quasi-static scenario. An important special case that significantly facilitates dynamic shakedown analysis, and which will be exploited in this work, is when the forcing functions are infinite and periodic [30,42–44].

The aim of this paper is to define a framework based on the aforementioned concept of dynamic shakedown that can be used for efficiently assessing the shakedown limit behavior of uncertain elastic perfectly plastic structures subject to long duration stochastic wind loads.

2. The dynamic shakedown problem for elastic perfectly plastic frames

2.1. Mechanical model

In order to introduce the formulation of interest to this study, it is first convenient to consider a discrete elastic perfectly plastic plane frame defined by n_b Euler–Bernoulli beam elements and n_N free nodes in a small displacement and deformation regime (the generalization to 3D frames is immediate). Consider indicating with $\boldsymbol{u}$ and $\boldsymbol{F}$ the vectors of dimensions n_f collecting the displacements and external loads of the free nodes, and with $\boldsymbol{q}$ and $\boldsymbol{Q}$ the vectors of dimensions n_d collecting the generalized strains and stresses where $n_f = 3 \cdot n_N$ and represents the total number of free degrees of freedom of the system while $n_d = 6 \cdot n_b$ represents the total number of stress and strain parameters at the ends of each beam element. The equilibrium of the aforementioned system can be expressed as:

$$\boldsymbol{C}^T\boldsymbol{Q} = \boldsymbol{F} - \boldsymbol{M}\ddot{\boldsymbol{u}} - \boldsymbol{V}\dot{\boldsymbol{u}} \tag{1}$$

where $\boldsymbol{C}^T$ is the equilibrium matrix while $\boldsymbol{M}$ and $\boldsymbol{V}$ are the mass and damping matrices. Furthermore, in Eq. (1) and in what follows, the over-dot indicates the time derivative while $(\cdot)^T$ indicates the transpose of the relevant quantity. Geometric compatibility between strains and displacements of the nodes can be imposed through:

$$\boldsymbol{q} = \boldsymbol{C}\boldsymbol{u} \tag{2}$$

where $\boldsymbol{q}$ represents the sum of an elastic ($\boldsymbol{e}$) and a plastic ($\boldsymbol{p}$) strain:

$$\boldsymbol{q} = \boldsymbol{e} + \boldsymbol{p} \tag{3}$$

while $\boldsymbol{C}$ is a compatibility matrix depending only on the geometry of the system. The elasticity equations for this system may be expressed as:

$$\boldsymbol{Q} = \boldsymbol{D}\boldsymbol{e} + \boldsymbol{Q}^* \tag{4}$$

where $\boldsymbol{D}$ is the block diagonal matrix containing the elastic stiffness matrices of the n_b beam elements defining the structure while $\boldsymbol{Q}^*$ is the vector collecting the perfectly clamped element generalized stresses.

The generalized stresses at each cross section of the structure cannot lie outside of the yield surface, therefore the vector $\boldsymbol{Q}$ must respect the following inequality:

$$\boldsymbol{\varphi} = \boldsymbol{N}^T\boldsymbol{Q} - \boldsymbol{R} \leqslant \boldsymbol{0} \tag{5}$$

where $\boldsymbol{\varphi}$ is the piece-wise linearized yield vector, $\boldsymbol{N}$ is a block diagonal matrix of unit external normals to the piece-wise linear convex yield surface while $\boldsymbol{R}$ is the plastic resistance vector. When at least one of inequalities of Eq. (5) is an equality, plastic strain can occur according to the following plastic flow rule:

$$\dot{\boldsymbol{p}} = \boldsymbol{N}\dot{\boldsymbol{\lambda}}, \quad \dot{\boldsymbol{\lambda}} \geqslant \boldsymbol{0}, \quad \boldsymbol{\varphi}^T\dot{\boldsymbol{\lambda}} = \boldsymbol{0}, \quad \dot{\boldsymbol{\varphi}}^T\dot{\boldsymbol{\lambda}} = \boldsymbol{0} \tag{6}$$

in which $\boldsymbol{\lambda}$ represents the vector of plastic multipliers. Eqs. (1)–(6) together with the initial conditions

$$\boldsymbol{u} = \boldsymbol{u}_0, \quad \dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}_0, \quad \boldsymbol{p} = \boldsymbol{p}_0, \quad \text{for } t = 0 \tag{7}$$

govern the dynamic analysis problem of elastic perfectly plastic plane frames. It is worth noting that the external stiffness matrix of the plane frame is given by $\boldsymbol{K} = \boldsymbol{C}^T\boldsymbol{D}\boldsymbol{C}$. This problem is usually solved by means of a step-by-step procedure set in a deterministic environment. To the authors' knowledge, no exact solution exists.

When the response of an elasto-plastic structure subject to dynamic loading becomes purely elastic, after a first phase of finite duration in which some plastic deformations are produced, the structure is said to have adapted to an elastic state and "dynamic shakedown" has occurred. In other words, a finite field of time-independent plastic strains has formed that allows the structure to respond in a purely elastic regime. The term shakedown implies the finite nature of the plastic deformations that in general are to be considered modest even though their exact amount is unknown as is the exact time at which the plastic phase ends and the purely elastic one begins. If the structural response exceeds the shakedown limit, the structure is exposed to a sort of inadaptation collapse, characterized by the uncontrolled growth of the plastic deformation during the load history with the possibility of failure due to the excessive accumulation of plastic strains (incremental collapse), or by inverting plastic strains in a cyclic loading scenario, with the possibility of failure by fatigue (alternating plasticity collapse).

2.2. Dynamic shakedown

A criterion for dynamic shakedown, for a structure subject to a fully specified loading history from $t = 0$ to $t = +\infty$, was given first by Ceradini [41,43] and is as follows: a necessary and sufficient condition for dynamic shakedown is that there exists a finite time $r \geqslant 0$, and some initial conditions in terms of displacements $\boldsymbol{u}_0^*$, velocities $\dot{\boldsymbol{u}}_0^*$ and plastic strains $\boldsymbol{p}_0^*$, such that the purely elastic stress response (fictitious) to the given load history with these initial conditions $\hat{\boldsymbol{Q}}(t)$ proves to be inside the yield surface at any subsequent time $t \geqslant r$:

$$\boldsymbol{\varphi} = \boldsymbol{N}^T\hat{\boldsymbol{Q}}(t) - \boldsymbol{R} < \boldsymbol{0}, \quad \forall t \geqslant r. \tag{8}$$