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Explicit reliability sensitivities of linear structures with interval uncertainties under stationary stochastic excitation

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Reliability sensitivity evaluation of randomly excited linear structures with uncertain parameters is addressed. The excitation is modeled as a stationary Gaussian random process. Uncertainty affecting structural parameters is handled within a non-probabilistic framework by applying the interval model. Under the assumption of independent up-crossings of a specified threshold, a procedure for the analytical derivation of interval reliability sensitivity is presented. The key idea is to perform stochastic analysis in the frequency domain by applying the *improved interval analysis via extra unitary interval* in conjunction with a novel series expansion of the inverse of an interval matrix with modifications, herein called *Interval Rational Series Expansion* (*IRSE*). This approach yields approximate explicit expressions of interval response statistics and structural reliability along with their sensitivities with respect to the uncertain parameters. Reliability sensitivities provide useful information to increase the safety level and can be also exploited in the context of *first-order interval Taylor series expansion* to estimate the bounds of interval reliability when slight uncertainties are involved. A wind-excited truss structure with interval stiffness properties is analyzed to show the effectiveness of the proposed procedure.

1. Introduction

It is now widely recognized that uncertainties affecting both structural parameters and external loads (see e.g., [1–3]) need to be included in structural reliability assessment to obtain credible estimates of failure probability. On one hand, unavoidable uncertainties existing in the design process, such as fluctuations in material properties or variations caused by manufacturing and assembly techniques, may cause serious changes in the performance and reliability of structural systems. On the other hand, real engineering structures are designed to fulfill prescribed safety requirements under environmental loads, such as earthquake ground motion, sea waves, gusty winds etc., which have an intrinsic random nature. As a consequence, structural safety evaluation has been traditionally carried out in the realm of random vibration theory.

Uncertainties affecting structural parameters, such as geometrical and mechanical properties, are commonly described within a probabilistic framework as random variables or random fields with assigned probability density function (*PDF*). However, the credibility of traditional probabilistic reliability methods relies on the availability of sufficient data to describe accurately the probabilistic distribution of the uncertain variables, especially in the tails. Indeed, reliability estimates are very sensitive to small variations of the assumed probabilistic models [4,5]. In the last decades, several non-probabilistic approaches [6], such as convex models, fuzzy set theory or interval models, have been developed for handling uncertainties described by fragmentary or incomplete data. Ben-Haim [4] first introduced a non-probabilistic concept of reliability claiming that: "Probabilistically, a system is reliable if the probability of unacceptable behavior is sufficiently low. In the non-probabilistic formulation of reliability, a system is reliable if the range of performance fluctuations is acceptably small". Since then, researchers focused the attention on the definition of new non-probabilistic reliability measures alternative to the traditional ones (see e.g., [7–12]).

A very useful tool to increase the safety level of a structural system or identify the crucial design parameters is represented by reliability sensitivity analysis which provides information on the variation in reliability due to a change in structural parameters. The knowledge of reliability sensitivities with respect to design parameters plays a fundamental role in the context of

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reliability-based optimization procedures (see e.g., [13]). In the case of structural systems exhibiting random uncertain parameters, reliability sensitivity analysis is used to find the rate of change in the probability of failure due to changes in distribution characteristics of random input, such as the means and standard deviations (see e.g., [14,15]). In general, reliability sensitivity analysis involves the evaluation of the partial derivatives of the failure probability (or reliability) with respect to deterministic design variables or distribution parameters of uncertain variables. In the literature, several contributions have been devoted to efficient evaluation of reliability sensitivity. A desirable feature in reliability sensitivity estimation is the capability of handling a large number of design variables and/or uncertain parameters [14,16–20].

To the best of the authors' knowledge, not much research effort has been devoted to the reliability sensitivity analysis of MDOF structural systems under stochastic excitation. Valdebenito et al. [21] proposed a procedure for estimating the reliability sensitivity of linear structures under Gaussian stochastic excitation. Chaudhuri and Chakraborty [22] derived analytical sensitivities of time-varying reliability with respect to design variables for structures under non-stationary earthquake ground motion. Recently, Greco and Trentadue [23] proposed an analytical method in the time domain for evaluating response covariance and reliability sensitivities of linear structures under non-stationary modulated filtered white noise process.

Studies on reliability analysis of randomly excited structures with uncertain parameters [13,24,25] have been carried out mainly within the probabilistic framework focusing on the estimation of the statistics of the failure probability (or reliability). Much less attention has been devoted to reliability sensitivity analysis of structures under stochastic excitation especially in presence of uncertain parameters with incomplete information.

To fill this gap, the present contribution deals with reliability sensitivity evaluation of linear structural systems with uncertain-but-bounded parameters subjected to stationary Gaussian random excitation. Due to the uncertainty affecting structural parameters, the reliability turns out to be an interval function and structural performance ranges between lower and upper bounds. The main objective of this study is to develop a procedure for deriving approximate explicit expressions of the interval reliability function and the associated interval sensitivities with respect to the uncertain parameters. According to the definition introduced by Moens and Vandepitte [26], the interval reliability sensitivities provide a measure of the degree of influence of the width of each input interval on the width of the interval reliability. The failure is assumed here to occur as the random process modeling the response quantity of interest (e.g., displacement, stress or strain at a critical point) firstly exceeds a safe domain within a specified time interval $[0,T]$. The proposed procedure is developed under the assumption that consecutive crossings of a specified threshold are statistically independent events so as to constitute approximately a Poisson process [27–29]. As known, in this case reliability evaluation involves the zero- and second-order spectral moments of a selected stationary response process. The underlying idea here is to derive the interval spectral moments of the stochastic response process and the corresponding interval reliability function in approximate closed-form by applying an approach recently proposed by the authors [30,31]. Basically, this approach relies on the use of the so-called *Interval Rational Series Expansion* (*IRSE*) [30,32] in conjunction with the *improved interval analysis via extra unitary interval* [30,33], introduced to limit the overestimation due to the *dependency phenomenon* [34,35] affecting the *"ordinary" or classical interval analysis* [36]. The *IRSE* is an alternative explicit form of the Neumann series expansion [37,38] for the inverse of an interval matrix with small rank r modifications. Once an analytical approximation of the interval reliability function is obtained, its interval sensitivities with respect to the uncertain parameters can be derived straightforwardly by direct differentiation. For small deviation amplitudes of the interval parameters, the interval sensitivities can be exploited in the context of *first-order interval Taylor series expansion* [39] to obtain accurate estimates of the upper and lower bounds of the interval reliability. It has to be mentioned that, for the sake of simplicity, in the present study the mass matrix is assumed deterministic. However, the proposed procedure can be readily extended to handle also mass uncertainties [30]. Furthermore, it is worth remarking that in the paper reliability analysis is performed in the nodal space, but classical modal analysis can also be applied [31].

An illustrative example concerning a wind-excited truss structure with interval Young's moduli is presented. Numerical results demonstrated the capability of the presented procedure to analyze the effects and relative importance of stiffness fluctuations on the safety level of the structure.

2. Improved interval analysis via extra unitary interval: basic definitions

The interval model is a widely used non-probabilistic approach to handle uncertainties occurring in engineering problems. This model, stemming from the interval analysis [36], turns out to be very useful when only the range of variability of the uncertain parameters is available. The aim of this section is to introduce some basic notations and definitions of the so-called *improved interval analysis via extra unitary interval*, recently proposed [30,33] to overcome the main limitations of the *"ordinary" or classical interval analysis* in structural engineering applications.

Denoting by $\mathbb{IR}$ the set of all closed real interval numbers, let $\boldsymbol{\alpha}^I \triangleq [\underline{\boldsymbol{\alpha}}, \bar{\boldsymbol{\alpha}}] \in \mathbb{IR}^r$ be a bounded set-interval vector of real numbers, such that $\underline{\boldsymbol{\alpha}} \le \boldsymbol{\alpha} \le \bar{\boldsymbol{\alpha}}$. In the following the apex I denotes interval variable while the symbols $\underline{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\alpha}}$ denote the lower bound (*LB*) and upper bound (*UB*) vectors. Since the real numbers α_i, collected into the vector $\boldsymbol{\alpha}$, are bounded by intervals, all mathematical derivations involving α_i should be performed by means of the *classical interval analysis* [36]. Unfortunately, the *classical interval analysis* suffers from the so-called *dependency phenomenon* [34,35] which often leads to an overestimation of the interval solution width that could be catastrophic from an engineering point of view. This occurs when an expression contains multiple instances of one or more interval variables. Indeed, the classical interval arithmetic operations assume that the operand interval numbers are independent. When the operands are partially dependent on each other, not all combinations of values in the given intervals will be valid and the exact result interval will generally be smaller than the one produced by the formulas.

To limit the effects of the dependency phenomenon, the so-called *generalized interval analysis* [40] and the *affine arithmetic* [41,42] have been introduced in the literature. In these formulations, each intermediate result is represented by a linear function with a small remainder interval [43]. Following the philosophy of the *affine arithmetic*, within the framework of structural analysis, the so-called *improved interval analysis via extra unitary interval* has been proposed [30,33]. The key feature of this approach is the introduction of the *extra unitary interval* (*EUI*), $\hat{e}_i^I \triangleq [-1,+1]$, $(i = 1,2,\ldots,r)$, defined in such a way that the following properties hold:

$$
\begin{aligned}
&\hat{e}_i^I - \hat{e}_i^I = 0; \quad \hat{e}_i^I \times \hat{e}_i^I = \left(\hat{e}_i^I\right)^2 = [1,1]; \quad \hat{e}_i^I/\hat{e}_i^I = [1,1] \\
&\hat{e}_i^I \times \hat{e}_j^I = [-1,+1],\ i \ne j; \quad x_i\hat{e}_i^I \pm y_i\hat{e}_i^I = (x_i \pm y_i)\hat{e}_i^I; \\
&x_i\hat{e}_i^I \times y_i\hat{e}_i^I = x_i y_i \left(\hat{e}_i^I\right)^2 = x_i y_i [1,1].
\end{aligned}
\tag{1a – f}
$$

In these equations, $[1,1] = 1$ is the so-called unitary *thin interval*. It is useful to remember that a thin interval occurs when $\underline{x} = \bar{x}$ and it is