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A novel Bayesian approach for structural model updating utilizing statistical modal information from multiple setups

Wang-Ji Yan *, Lambros S. Katafygiotis

Hong Kong University of Science and Technology, Department of Civil and Environmental Engineering, Clear Water Bay, Kowloon, Hong Kong

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ABSTRACT

In this paper, a fast Bayesian methodology is presented for structural model updating utilizing modal information from multiple setups. A two-stage fast Bayesian spectral density approach formulated recently is firstly employed to identify the most probable modal properties as well as their uncertainties. The model updating problem is then formulated as one minimizing an objective function, which can incorporate statistical information about local mode shape components corresponding to different setups automatically, without prior assembling or processing. A fast analytic-iterative scheme is proposed to efficiently compute the optimal parameters so as to resolve the computational burden required for optimizing the objective function numerically. The posterior uncertainty of the model parameters can also be derived analytically and the computational difficulty in estimating the inverse of the high dimensional Hessian matrix required for specifying the covariance matrix is also properly tackled. The efficiency and accuracy of all these methodologies are verified by numerical examples.

1. Introduction

Vibration-based damage detection has received enormous amount of attention since it is able to provide a global approach to evaluate the structural condition [1]. Over the past few decades, a wide variety of structural damage detection methods has been developed, which culminates in various papers [2,3]. Based on their dependency or not on an analytical model, vibration-based damage assessment methods are usually divided into model-free and model-dependent ones [4]. The model free methods might be able to detect and locate structural damage, but they are not likely to establish structural damage severity. On the contrary, the model-dependent approaches are expected to achieve the goal of determining the existence of structural damage, establishing the damage location, and the severity of damage. The common principle for the model-based approaches is to determine the structural model parameters before and after a possible damage from measured dynamic responses. As a result, they usually require solving the inverse problem given some measured data by employing structural model updating methodologies [5].

Most of the model-dependent approaches fail to accommodate various kinds of uncertainties. For example, the structures under consideration are usually assumed to be well characterized by initial analytical models, while a sufficiently large amount of measurements with low level of noise are assumed to be available for model updating. The approaches under these assumptions are more likely to produce poor results when applied in realistic scenarios in practice. Therefore, the problem on how to treat the uncertainties explicitly arises and is recognized as an important issue that needs to be addressed [6]. In this regard, stochastic model updating has gained increasing popularity in both theory development and practical applications. The minimum variance method [7], random matrix theory method [8], covariance matrix adjustment method [9], Interval model updating approach [10,11], and perturbation method [12] are all good examples of model updating methods dealing with various kinds of uncertainties. Another novel school of thoughts for statistical inference and uncertainty quantification is the Bayesian framework, since it is capable of finding the plausible structural damage extents as well as their probabilities given a model of the structural system and the measured data [13]. Using the Bayesian statistical framework, a number of probabilistic model updating approaches has been proposed based on identified modal parameter data sets [5,14–19]. Though significant progress has been achieved, full-fledged applications of Bayesian approaches in structural probabilistic model updating are still in their infancy. In particular, it is highly non-trivial to propagate the various uncertainties when updating the parameters of the finite element model, a process which is usually very demanding in terms of

* Corresponding author. Tel.: +852 53730845.
E-mail addresses: civilyanwj@gmail.com (W.-J. Yan), lambros@ust.hk (L.S. Katafygiotis).

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computational effort. In conventional Bayesian model updating procedures, the Bayesian spectral density approach [20] and Bayesian FFT approach [21] are common choices for obtaining the most probable modal properties and their uncertainties for given measured data and modeling assumptions. Unfortunately, computational difficulty has severely hindered their applications even for a very moderate number of measured dofs. Therefore, more efficient ambient modal analysis strategies need to be developed to determine the posterior statistics of modal parameters.

In full-scale ambient tests, multiple setups sharing some reference sensors are usually employed due to practical difficulties in deploying a very large number of sensors or due to limited instrumentation budget [22]. In some cases when computers available have limited memory space and computational capacity or when a distributed computation strategy is to be implemented under the environment of a wireless sensor network, the huge amount of data acquired synchronously by a single setup has also to be divided into multiple setups and processed individually for each setup. As a result, a group of natural frequencies and local mode shape components corresponding to different setups is usually available. The uncertainties of modal parameters vary across different setups due to the variability of signal quality under ambient vibration conditions [23]. Therefore, the question on how to fuse the posterior statistics of local modal information obtained from different setups becomes an important issue in statistical model updating. Few model updating approaches available are able to incorporate statistical modal information from different setups using an automated procedure. Conventionally, global mode shapes ought to be assembled from different local mode shapes prior to structural model updating. Most of the model updating approaches can only make use of the optimal global mode shapes and fail to utilize their uncertainty information. For these reasons, there is still significant room to develop efficient Bayesian model updating approaches so that the local modal information with varying degrees of uncertainty from different setups can be rationally incorporated in an efficient and automated manner.

Structural model updating incorporating in a consistent manner statistical modal information obtained from different setups is not trivial. Several critical issues may arise in real implementation. In general, local mode shape components estimated from different setups may have different scaling factors as they are normalized individually [22]. Also, more than one reference dof is required when the reference dof has no significant frequency response in some of the modes of interest. Moreover, in many cases there is no single fixed reference sensor that is shared by all measured setups. In this study, an advanced Bayesian statistical algorithm that is able to treat the critical aforementioned difficulties is proposed. Theoretical and computational issues of the Bayesian model updating problem are well addressed. The proposed method is able to incorporate statistical modal information identified from multiple setups into the model updating procedure automatically without prior mode shape assembling or processing.

The manuscript of this study is organized as follows. Section 2 introduces a two-stage fast Bayesian spectral density approach proposed recently to obtain the most probable modal parameters as well as their uncertainties. The Bayesian model updating problem can then be formulated as one minimizing an objective function in Section 3. Section 4 presents a fast iterative scheme so as to resolve the computational burden required for optimizing the objective function numerically. In Section 5, the Hessian matrix of the identified parameters is derived analytically. Moreover, the computational difficulty in estimating the inverse of the high dimensional Hessian matrix required for specifying the covariance matrix of the model parameters is also properly addressed. Finally, numerical examples are presented to illustrate the efficiency of the proposed model updating method.

2. Posterior statistics of modal information obtained from multiple setups

Recently, a breakthrough was made by Au [24–26] in the field of ambient modal analysis to address the computational challenges of the conventional Bayesian FFT approach [21] allowing to drastically increase the efficiency in computing the most probable values as well as the corresponding posterior covariance matrix. Motivated by the conventional Bayesian spectral density approach [20] and the fast Bayesian FFT approach, a two-stage fast Bayesian spectral density approach was formulated more recently, and will be employed in this study for ambient modal analysis. The original formulation can be found in [27,28] and only the main concept is reviewed in this section.

Assume that there are n_s sets of independent and identically distributed time histories for n_o measured dofs. Fast Fourier Transform (FFT) of the response history $\mathbf{y}_j$ at frequency point f_k is denoted as $\mathbf{Y}_j(k)$ which approximately follows a complex normal distribution. The covariance matrix of $\mathbf{Y}_j(k)$ is denoted by $\mathbf{C}_k(\vartheta)$, a function of the modal parameters ϑ to be identified. Then the sum of sets of spectral density matrix estimators $\mathbf{S}_k^{\mathbf{sum}} = \sum_{j=1}^{n_s}\mathbf{Y}_j(k)\mathbf{Y}_j^*(k)$ follows a central complex Wishart distribution. Furthermore, it has been proved that the trace of $\mathbf{S}_k^{\mathbf{sum}}$ denoted by $tr(\mathbf{S}_k^{\mathbf{sum}})$ asymptotically follows a normal distribution when n_s is large. The spectral density set formed over the frequency band $[f_{k_1}, f_{k_2}]$ is employed for ambient modal analysis. There are two possible cases for each specified frequency band, i.e., the case of separated modes with a single mode to be identified, and the case of closely spaced modes with multiple modes to be identified. For both cases, it has been proved that the interaction between spectrum variables (frequency, damping ratio as well as the spectral density of modal excitation and prediction error) and the spatial variables (mode shapes) can be decoupled so that they can be identified separately.

In the first stage, the spectrum variables can be separated from the full set of parameters with the help of 'fast Bayesian spectral trace approach' (FBSTA) by employing the statistical properties of the sum of auto-spectral densities corresponding to different measured dofs. As a result, the optimal spectrum variables as well as the covariance matrix can be identified by manipulating the following 'negative log-likelihood function' (NLLF),

$$L_1(\vartheta) = \frac{1}{2}\sum_{k=k_1}^{k_2}\ln\left(2\pi n_s tr\left(\Re\left(\mathbf{C}_k^2(\vartheta)\right)\right)\right) + \sum_{k=k_1}^{k_2}\frac{\left[tr\left(\mathbf{S}_k^{\mathbf{sum}}\right) - n_s tr(\mathbf{C}_k(\vartheta))\right]^2}{2n_s tr\left(\Re\left(\mathbf{C}_k^2(\vartheta)\right)\right)} \quad (1)$$

where $\Re(\cdot)$ denote the real part of a complex matrix. Once the spectrum variables are extracted, the mode shapes as well as their uncertainties can be identified in the second stage with the help of 'fast Bayesian spectral density approach' (FBSDA). The most probable mode shapes as well as their covariance matrix can be obtained by manipulating the following NLLF,

$$L_2(\vartheta) = n_s\sum_{k=k_1}^{k_2}\ln|\mathbf{C}_k(\vartheta)| + \sum_{k=k_1}^{k_2}tr\left(\mathbf{C}_k^{-1}(\vartheta)\mathbf{S}_k^{\mathbf{sum}}\right) \quad (2)$$

where $tr(\cdot)$ and $|\cdot|$ denote the trace and determinant of a matrix, respectively. As a result, the updated PDF of the identified parameters can be well-approximated by a Gaussian PDF centered at the optimal parameters with covariance matrix equal to the inverse of the Hessian matrix of the NLLFs.

As mentioned in Section 1, the dofs of interest are usually measured in different setups with common 'reference' dofs present across different setups. In the first stage, the auto-spectral densities