Structural Safety 53 (2015) 36–43

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

A stochastic framework to model deterioration in engineering systems

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ARTICLE INFO

Article history:
Received 9 August 2013
Received in revised form 26 November 2014
Accepted 2 December 2014
Available online 21 January 2015

Keywords:
Stochastic framework
Deterioration
Time to failure

ABSTRACT

Almost every engineering system deteriorates over time due to exposure to extreme conditions and during routine use. Deterioration is a serious concern in engineering because it can considerably reduce the life and reliability of systems. In this work, we develop a novel general stochastic framework to model the deterioration process. We model the deterioration process as a combination of shock and gradual process. We account for the effects of deterioration on both capacity of a system and the imposed demands on the system. The framework accounts for two types of failure; demand exceeding capacity and accumulated damage exceeding allowable limit. An accurate and computationally efficient semi-analytical solution and an approximate solution are proposed to estimate the time to failure. We illustrate the steps in the proposed framework using two numerical examples. The proposed framework can be used to improve the reliability of systems and to optimize their performance.

1. Introduction

Engineering systems deteriorate while in service due to exposure to extreme conditions (e.g., excessive loading and harsh environment) and routine use. Deterioration is a serious concern because it can considerably reduce the service life and reliability of systems. Moreover, the process of deterioration is highly unpredictable and often invisible. Therefore, a design of systems that accounts for the various uncertainties in the deterioration processes is of utmost importance.

There is a considerable amount of literature available on the modeling of reliability of deteriorating systems. Studies have focused on estimating how the reliability of a deteriorating system changes with time. In this regard, researchers have used existing models or laboratory data to predict the state of a system (e.g., size of a crack or cross-sectional area of reinforcement steel lost to corrosion) at any given time and computed the time-variant reliability analysis for systems [1–5]. Estimating the change in reliability of a system with time is useful in designing and operating a system. However, other important variables also need to be estimated such as time to failure under random occurrences of loads and the number of loads until failure. For estimating these variables, stochastic modeling of the load arrival process, the deterioration process and also the dependencies between the two processes need to be modeled. In this regard, several researchers have proposed stochastic models [6–15]. In this paper, we propose a framework for stochastic modeling of deteriorating systems that addresses some of the modeling issues in the existing literature.

Broadly speaking, two ways of modeling are prevalent in the existing literature on stochastic modeling of deteriorating systems based on the focus on the type of failure. The two possible types of failures of a system are: (1) failure occurs when a cumulative damage or a deterioration process exceeds a permissible limit [6,10,12–15] and (2) failure occurs when an engineering demand parameter exceeds the corresponding capacity [7–9]. In the former, the deterioration process usually is represented using a monotonic continuous stochastic process which is then used to predict the time to failure and time-variant reliability. In the latter, theoretically, the effect of deterioration on both capacity and demand should be modeled. To model these phenomena, at least a monotonic continuous stochastic process and a point process is required. However, researchers in this area have often simplified the modeling process by only considering the effect of deterioration on capacity and not on the demand and by considering the capacity deterioration process independent of demands. In this paper, we propose a framework to account for the effect of deterioration on both capacity and demand of a system. Furthermore, we also account for the possibility of both types of failures because a complex system may experience any of the two possible failures.

This paper is organized into six sections. After this introduction, Section 2 describes the general deterioration process and discusses issues related to the stochastic modeling of deterioration

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http://dx.doi.org/10.1016/j.strusafe.2014.12.001
0167-4730/

processes. Sections 3 and 4 present the framework to model the deterioration process along with two possible solutions strategies. Section 5 presents three numerical examples to illustrate the computation of various quantities of interest. Finally, the last section presents a summary and discussions on this research.

2. The deterioration process

Fig. 1 illustrates a general deterioration process and the failure of a deteriorating system. During the service life, a system is subject to a sequence of loads $\{S_{t_n}\}$ at times $\{t_n\}$ $(n = 1, 2, \ldots)$. At time $\tau = t_n$, the external load S_{t_n} imposes a demand $D_{t_n} = D(\mathbf{x}_{t_n}, S_{t_n})$ on the system, where $\mathbf{x}_{t_n}$ represents the properties of the system at time t_n and $D(\mathbf{x}_{t_n}, S_{t_n})$ is a function of $\mathbf{x}_{t_n}$ and S_{t_n}. It is shown that the system experiences a shock deterioration at $\tau = t_{n-1}$ and t_n, and gradual deterioration in the interval (t_{n-1}, t_n) and for $\tau > t_n$. The capacity C_τ gradually changes from $C_{t^+_{n-1}}$ to $C_{t^-_n}$ and instantaneously changes from $C_{t^-_{n-1}}$ to $C_{t^+_{n-1}}$ and $C_{t^-_n}$ to $C_{t^+_n}$ (where t^-_i and t^+_i are the time instants immediately before and after t_i). The failure of a system can be of two types: (1) D_{t_n} exceeds $C_{t^-_n}$ i.e. $(C_{t^-_n} - D_{t_n}) < 0$; (2) $W_\tau > w_a$, where W_τ is an index of the level of deterioration in the system at time τ which is a function of the change in capacity and demand characteristics of the system and w_a is a predetermined permissible level of deterioration. It must be noted that the first type of failure event at any t_n also implies that the second type of failure occurs at t_n. However, the converse need not be true. The second type of failure can occur at any time even if D_{t_n} is not acting on the system.

In this section, we discuss the important issues that need to be addressed in modeling the process of deterioration and failure of a system. They are:

1. *Modeling the effect of deterioration on capacity*. The issue of reduction in capacity due to deterioration has been widely acknowledged and addressed in the models for deteriorating systems. The reduction in capacity has been modeled in the past using random and deterministic functions of time [7,10–12]. In this paper, we account for the deterioration in capacity using a stochastic process which is a combination of a deterministic function and a compound Poisson process.
2. *Modeling the dependence between the deterioration process and demands*. The shock deterioration process should be modeled as dependent on the process $\{D_{t_n}\}$. This is because the deterioration in a system at time t_n is a result of D_{t_n}. Furthermore, the process $\{D_{t_n}\}$ should be modeled as a process dependent on the state of the system. This is because $\mathbf{x}_t$ is dependent on time and therefore the probability distribution of the demand can change with time. This inter-dependence between the deterioration process and $\{D_{t_n}\}$ has not been addressed adequately in literature. In this paper, we model the effect of deterioration on $\{D_{t_n}\}$ using a combination of a stationary process that represents demands on the undeteriorated system and a monotonically increasing stochastic process to incorporate the effect of deterioration on demands.
3. *Modeling the combination of shock and gradual deterioration process*. Most engineering systems experience both shock and gradual deterioration. In literature, several studies have modeled both shock and gradual deterioration [6,9–12,14,15]. A major challenge in this aspect is modeling the interaction between shock and gradual deterioration which are usually modeled as independent of each other. In this paper, we also model the two processes independent of each other where we use a deterministic function of time to model gradual deterioration and a compound Poisson process to model the shock deterioration. While this framework can be extended to consider a stochastic process for gradual deterioration, we have considered a deterministic function for simplicity. We acknowledge that researchers have in the past used Gamma process for modeling gradual deterioration [11,12,14]. However, in our view, the random fluctuations of the rate of gradual deterioration have a small effect on the system compared to the overall trend of gradual deterioration over a long period of time which can be modeled with a nonlinear deterministic function.
4. *Modeling the failure event*. In the existing literature, generally only one failure type is considered. However, in general a system can experience either type of the two failures in its life time as described earlier. Noortwijk et al. [11] considered both types of failures but used simplifying assumptions like mutual independence between the deterioration process and $\{D_{t_n}\}$. However, the study did not consider the effect of deterioration on demand.
5. *Proposing computationally efficient solution*. Some studies (e.g., [15]) use basic Monte Carlo simulations of deterioration process, loading events and failures of the system to estimate the time to failure of a deteriorating system. Basic Monte Carlo simulations can be used to compute the expectations of time to failure, but it can be computationally expensive to compute its probability distributions using this method engineering systems with very low failure probabilities. In this paper, we propose simulation based approach that requires less number of simulations to compute the probability distribution of time to failure.

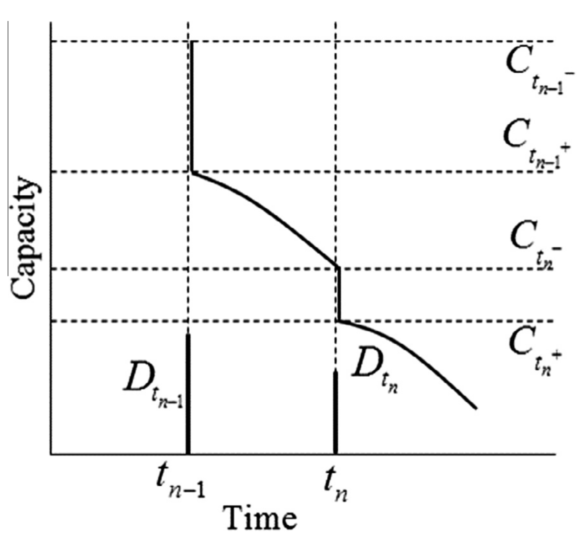

Fig. 1. Effect of deterioration process on capacity.

3. Proposed framework

Deterioration is a complex process especially when the loads and the deterioration process are dependent on each other. In order to develop a framework that can address the previously discussed issues with respect to modeling a deterioration process, we make the following assumptions:

1. The total effect of shock and gradual deterioration process on the capacity and demand of a system is the sum of the effects of the individual deterioration processes.
2. The shock and the gradual deterioration process are independent of each other.
3. The shock deterioration process is independent of time and is composed of statistically independent and identically distributed (*s.i.i.d*) shocks, where each shock is dependent on the corresponding demand on the system.