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On the Pareto optimality of variance reduction simulation techniques in structural reliability

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ABSTRACT

Estimation of low failure probabilities in high dimensional structural reliability problems generally involves a trade-off between computational effort and accuracy of the estimate, whether efficient sampling techniques have been employed or not. While a substantial effort continues to be made by the community to develop and benchmark new and efficient sampling schemes, the limits of performance of a given algorithm, e.g., what is the best attainable accuracy of the method for a fixed computational effort and if that is good enough, have not received comparable attention. However, such insights could prove valuable in making the right choice in solving a computationally demanding reliability problem. In a multi-objective stochastic optimization formulation, these questions yield the so-called Pareto front or the set of non-dominated solutions: solutions that cannot be further improved without worsening at least one objective. Posteriori user defined preferences can then be applied to rank members of the Pareto set and obtain the best strategy. We take up two classes of variance-reducing algorithms – importance sampling (IS) and subset simulations (SS) – and apply them to a range of benchmarked reliability problems of various size and complexity to bring out the issue of optimality and trade-off between accuracy and effort. The design variables are variously of categorical, discrete as well as continuous types and the stochastic multi-objective optimization without recourse is solved using Genetic Algorithms. In each case, we ascertain the best possible accuracies that a given method can achieve and identify the corresponding design variables. We find that the proposal pdf does have an effect on the efficiency of SS, the FORM design point is not always the best sampling location in IS and setting the sensitivity parameter associated with Adaptive Importance Sampling at 0.5 does not guarantee optimal performance. In addition to this the benefits of using SS for high dimensional problems are reinforced. We also show that the Pareto fronts corresponding to different methods can intersect indicating that more is not always better and different solution techniques for the same problem may be required in different computational regimes.

1. Introduction

For a distributed structure with several potential critical locations and failure modes (such as shear and flexure), subject to time dependent loads and possessing time- and space-dependent material properties, the reliability function estimates the probability that the capacity, C, exceeds the demand, D, in each failure mode, at all locations and at all times that the structure is in service:

$$\mathrm{Rel}(t) = 1 - P_f(t) = P[C_j(\underline{x},\tau) \geqslant D_j(\underline{x},\tau), j \leqslant J, \forall \tau \in (0,t), \forall \underline{x} \in \Omega] \qquad (1)$$

where Ω is the set of critical locations of the structure, J is the total number of failure modes at each critical location, and t is total time horizon. Both capacity and demand of the structure are generally functions of space and time and constitute a multidimensional stochastic process.

The structural reliability problem in its most general formulation is thus infinite dimensional both in time and space which of course makes it computationally intractable; hence various levels of simplification are adopted. If there is only one critical location with only one failure mode, and demand and capacity are time invariant as well, we have the most basic formulation: a time-independent element level reliability problem which typically is described by a few basic variables and can be solved by elegant geometric techniques such as FORM and SORM. Monte Carlo simulation based techniques can also be adopted with ease. Time invariant problems with more than one failure mode and/or location can be modeled as a system reliability problem with appropriate unions and intersection of element level limit states

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and can still be tackled with FORM/SORM although with increasing approximation. For such problems, simulation based techniques, especially with some efficient sampling scheme, may appear more desirable.

When the time dependent nature of C and/or D of the critical element cannot be neglected, the next level of complexity in reliability problems involves condensing the time dimension to a finite number of discrete points by modeling the load as a stationary pulse process and the capacity as a non-random function of time. A higher level of complexity occurs when stationarity can no longer be assumed due either to non-stationary excitation or to stochastic degradation: the first passage into failure by the process C–D at the critical location may need to be solved by direct simulations in the time domain.

In addition to modeling temporal randomness, spatial randomness may need to taken into account for distributed problems. Random fields describing the spatially varying quantities are discretized according to the set of critical locations and the discretization scheme adopted to solve the problem as using a finite element formulation, and can involve local averaging, series approximations, interpolation etc. [1]. Spatial randomness thus increases the dimensionality of the reliability problem; it also affects the statistical dependence between safety margins at different locations both at the same instant and at different instants of time.

For most real life structures one thus finds a high dimensional reliability problem [2]. In addition to this, very low failure probabilities are typically associated with structures owing to the high level of safety that society has come to expect of structures. A brief discussion on acceptable failure probabilities of different types of structures can be found in Bhattacharya et al. [3].

When the number of random variables become large, an important issue is how much one can trust the reliability index obtained from analytical approximate methods like FORM. In general the optimization procedure associated with FORM becomes unmanageable in high dimensions and it is advisable that simulation techniques be used [4]. The computation time increases with the number of random variables and if gradient computations are done numerically, the number of limit state function calls is proportional to the number of random variables [5]. Adhikari [6] estimated the failure probability using asymptotic distributions and derived a modified beta (actual beta estimate does not give correct values) for such cases. Schueller et al. [7] suggested that for dimensions greater than 30 FORM yields inaccurate solutions.

Simulation based approaches to structural reliability computation offer a far greater flexibility and can address many of the shortcomings of analytical based methods. At the same time, simulations have their limitations in terms of speed, size and accuracy. After all, simulation based algorithms are basically numerical statistical sampling schemes, and can never be free of sampling errors. All pseudo random number generators (as opposed to true random bit generators that are accurate but very slow [8]) suffer from finite periods (although the Mersenne Twister algorithm has one of the longest periods [9]). The gap therefore, at any given point of time, between computational need and computational resource, i.e., between the grand problem that the community would like to solve and the problem it is able to tackle due to hardware and/or algorithmic limitations, has always existed. Naturally, then, continual efforts have been made by the community to invent clever and efficient simulation schemes [7,10–15].

While substantial effort continues to be made to develop and benchmark new and efficient sampling schemes, the limits of performance of a given algorithm, e.g., what is the best attainable accuracy of the method for a fixed computational effort and if that is good enough, have not received comparable attention. We believe that such inquiries could prove valuable in making the right choice in solving a computationally demanding reliability problem. We investigate, from a multi-objective stochastic optimization viewpoint, two classes of variance-reducing algorithms – importance sampling and subset simulations – in order to bring out the issue of optimality and trade-off between accuracy and effort.

Even though comparative studies have been undertaken in the past, no author has tried to formulate it as a multi-objective optimization problem to the best of our knowledge. Previous comparative studies have only looked at superiority of one method over the other [7,12,15]. In addition to a comparative study the present work gives us an idea about the optimal combination of parameters to be used for a given levels of computational resources. Posteriori preferences of the user can be used to select the optimum method along with the optimum design parameters for best performance but is outside the scope of this paper.

We determine for a set of benchmark problems in structural reliability the best accuracy that a method can achieve given a fixed computational effort. We find that some conventional wisdoms, such as IS should be centered on the FORM design point and SS is not affected by the choice of proposal pdf, may not have much merit. We also demonstrate that the Pareto fronts corresponding to different methods can intersect indicating that more is not always better and different solution techniques for the same problem may be required at different regimes.

The structure of the paper is as follows. The next two sections give a brief overview of variance reduction techniques in structural reliability and multi-objective stochastic optimization problems in engineering. We then demonstrate the concepts of design variables, random objectives, solutions of various ranks, and the Pareto front through a simple one dimensional reliability problem. Following this, six benchmark reliability problems, in order of increasing complexity, are taken up in detail.

2. Variance reduction techniques in estimating reliability

If the reliability problem given in Eq. (1) can be expressed in terms of a finite number of basic variables, $\mathbf{X}$, whose membership in the failure region F can be verified by a finite number of binary checks summarized by the indicator function I_F, the failure probability is:

$$P_f = P(\mathbf{X} \in F) = \int I_F(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = E[I_F(\mathbf{X})] \tag{2}$$

There are a number of ways of solving the above integral as we have discussed above. The most robust simulation based technique for estimating P_f in Eq. (2) is the basic Monte Carlo Simulation (MCS) [16]. However, while MCS gives an unbiased estimate of P_f, the coefficient of variation (c.o.v.) of the MCS estimator is $\sqrt{(1-P_f)/(P_f N)}$ which clearly shows the unfavourable relation between accuracy and effort involved in basic MCS.

2.1. Importance sampling and its variants

In order to overcome the problem of low efficiency associated with basic Monte Carlo techniques, a number of variance reduction techniques have been proposed over the years [17,18], the most widely used being Importance Sampling (IS) [16] whose basic idea is to carry out the simulations in a region which is considerably closer to the limit state:

$$P_f = P(X \in F) = \int I_F(x)[f_X(x)/h_v(x)]h_v(x)dx = E\left[I_F(v)\frac{f_X(v)}{h_v(v)}\right] \tag{3}$$

where $h_v(.)$ is the importance sampling density function which one would ideally like to centre on the point of maximum likelihood. If $h_v(.)$ is suitably chosen, one may generate a relatively large number