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Pre-posterior optimization of sequence of measurement and intervention actions under structural reliability constraint

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ABSTRACT

It is common to assess the condition of an existing infrastructure using reliability analysis. When, based on the available information, an existing structure has an estimated failure probability above the admissible level, the default solution often is to either strengthen or replace it. Even if this practice is safe, it may not be the most economical. In order to economically restore and improve our existing infrastructure, the engineering community needs to be able to assess the potential gains associated with reducing epistemic uncertainties using measurements, before opting for costly intervention actions, if they become necessary. This paper provides a pre-posterior analysis framework to (1) optimize sequences of actions minimizing the expected costs and satisfying reliability constraints and (2) quantify the potential gain of making measurements in existing structures. Illustrative examples show that when the failure probability estimated based on the present state of knowledge does not satisfy an admissible threshold, strengthening or replacement interventions can be sub-optimal first actions. The examples also show that significant savings can be achieved by reducing epistemic uncertainties.

1. Introduction

With increased awareness about the extent of deficiencies of existing infrastructures, the US National Academy of Engineering has identified restoration and improvement of urban infrastructure as one of the grand engineering challenges of the 21st century [1]. It is common to assess the condition of an existing infrastructure by reliability analysis using prior knowledge about capacities and demands. When an existing structure has an estimated failure probability above an admissible level, $p_{\mathcal{F}} > p_{\mathcal{F}}^{\{adm.\}}$, the default solution often is to perform a *structural intervention action*, such as strengthening or replacement. However, it is known that the prior information about capacities and demands of an existing structure is characterized by epistemic uncertainties. By gathering additional information, it is often possible to reduce these uncertainties and alter the failure probability estimate. Therefore, in order to assess the true condition of an existing infrastructure and economically restore and improve it, the engineering community needs to be able to estimate the potential gains associated with reducing epistemic uncertainties using *information gathering actions*, instead of directly opting for costly structural interventions based on findings from prior knowledge.

Uncertainties and their classification have received much attention from the scientific community, e.g., [2–4]. Uncertainties are most often classified as either *aleatory* or *epistemic*, depending on whether they are attributed to inherent variability or to lack of knowledge. According to this classification, epistemic uncertainties are reducible and aleatory uncertainties are not. Several researchers have noted that, during the design phase, the uncertainties in structural properties are inherently random and, therefore, aleatory in nature [4,5]. However, once the structure is constructed, the uncertainties in structural properties become epistemic in nature. In a sense, the constructed structure is viewed as a realization from a population of structures having the same design. Naturally, if we were able to precisely measure the properties (e.g., as-built dimensions, material constants, member capacities) of an existing structure, no uncertainties in these quantities would remain. Of course, it is not possible to accurately measure all structural properties. Nevertheless, any direct or indirect observations about these quantities can serve to reduce the corresponding epistemic uncertainties. Note that measuring a structural property may either increase or decrease the estimated failure probability, depending on the measurement outcome [5,6]. Section 3.1.1 presents considerations that this aspect requires during the planning of measurement actions.

Maintenance planning for structures has been addressed in previous research related to structural health monitoring, decision

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theory and reliability theory. For instance, Faber [5] proposed a general framework for assessment of existing structures based on reliability theory considering evidences obtained during inspection. Pozzi and Der Kiureghian [7] used the concept of value of information (VoI) [8] to quantify the value of measuring the evolution of structural performance as a support to maintenance interventions. In a similar way, Glisic et al. [9] used VoI to quantify, in economic terms, the impact of monitoring on decision making. Straub and Faber [10] used decision and VoI theory to build an adaptive decision framework for identifying inspection planning strategies that minimize maintenance costs. In their framework, inspections are performed in a sequence, and the decision to perform an inspection is based on the outcome of the previous inspection.

Engineering decision analysis can be made in three stages [11–13]: prior decision analysis, posterior decision analysis and pre-posterior decision analysis. This paper deals with *pre-posterior* decision analysis, where the planning of information gathering actions is made based on the prior probabilistic model of uncertainties. In this scheme, the consequences (e.g., costs) of the possible outcomes of measurement or other information gathering actions are weighed with their probabilities of occurrence. This approach to measurement actions planning is similar to what was proposed by Artstein and Wets as the *theory of sensors* [6]. Interested readers may also consult other relevant work performed in the field of maintenance-action optimization [14–17]. In the field of reliability-based optimization, Royset et al. [18–20] studied several aspects related to the design of new structures, notably optimal design under constraints. Der Kiureghian et al. [21] were among the firsts to study inverse reliability problems, where parameter values satisfying a reliability constraint are sought. More recently, Lehký and Novák [22] also approach this problem using a method based on Artificial neural network. Despite all these related aspects previously addressed in the literature, solving the problem posed in this paper requires further investigations related to the optimization of sequences of information gathering and intervention actions.

This paper presents a pre-posterior framework for optimizing sequences of actions minimizing the expected costs and satisfying reliability constraints for an existing structure. This framework is intended to: (1) provide optimized sequences of information gathering and intervention actions, and (2) quantify the potential gains of measuring structures instead of directly opting for costly strengthening and replacement interventions. The paper is organized in the following order: Section 2 presents the formulation for assessing the reliability of an existing structure, Section 3 presents the mathematical framework for the pre-posterior decision analysis for sequences of actions, and Section 4 presents illustrative applications of the proposed methodology.

2. Assessing the reliability of an existing structure

The safety and serviceability of an existing structure is usually assured by verifying that, given the available knowledge, the structure has a failure probability (complement of reliability) lower or equal to an admissible value, i.e., $p_{\mathcal{F}} \leqslant p_{\mathcal{F}}^{\{\text{adm.}\}}$. Let $\mathbf{V} = [V_1, V_2, \ldots, V_n]^{\mathrm{T}}$ denote the set of random variables defining the state of the structure and $f_{\mathbf{V}}(\mathbf{v})$ represent its joint probability density function (PDF). The failure probability is defined as

$$p_{\mathcal{F}} = \int_{\Omega} f_{\mathbf{V}}(\mathbf{v}) \mathrm{d}\mathbf{v} \tag{1}$$

where

$$\Omega \equiv \{\mathbf{v} | \cup_k \cap_{i \in C_k} G_i(\mathbf{v}) \leq 0\} \tag{2}$$

is the failure domain. This formulation is written in terms of unions of intersections of componental failure events. The *i*th component is defined in terms of a limit state function $G_i(\mathbf{V})$ with $\{G_i(\mathbf{V}) \leqslant 0\}$ indicating its failure. The union operation is over min cut sets $C_k, k = \{1, 2, \ldots\}$, where each min cut set represents a minimal set of components whose joint failure constitutes failure of the structure. The intersection operations are over components within each min cut set. Special cases of this formulation are series structural systems, when each min cut set has a single component, parallel structural systems, when there is only one cut set, and structural component, when there is only one min cut set with a single component. See Der Kiureghian [23] for more details about this formulation.

The limit-state functions $G_i(\mathbf{V})$ defining the component states are usually made up of sub-models representing component capacity and demand values. Such a sub-model typically has the form

$$R(\mathbf{X}, \epsilon) = \widehat{R}(\mathbf{X}) + \epsilon \tag{3}$$

where $\hat{R}(\mathbf{X})$ represents an idealized mathematical model and ϵ is the model error, which is usually considered to have the Normal distribution. The additive error model is based on an assumption of normality, which is usually satisfied by an appropriate transformation of the model, see [24]. Physics-based models of structural components are generally biased so that the mean of ϵ, μ_ϵ, can be nonzero. The standard deviation, σ_ϵ, represents a measure of quality of the model. The vector $\mathbf{V}$ collects random variables $\mathbf{X}$ and ϵ for all sub-models. In addition, it may include any uncertain parameters Θ involved in the definition of the distributions of $\mathbf{X}$ and ϵ for the various sub-models.

At the outset of our analysis, the PDF of $\mathbf{V}$ represents our prior state of knowledge about the structure and its future loads. We designate this by using the notation $f_{\mathbf{V}}^{\{0\}}(\mathbf{v})$. The corresponding estimate of the failure probability is denoted $p_{\mathcal{F}}^{\{0\}}$. If $p_{\mathcal{F}}^{\{0\}} \leqslant p_{\mathcal{F}}^{\{\text{adm.}\}}$, the reliability constraint ($p_{\mathcal{F}}^{\{\text{adm.}\}}$) is satisfied and no further action is necessary. When $p_{\mathcal{F}}^{\{0\}} > p_{\mathcal{F}}^{\{\text{adm.}\}}$, actions are necessary to reduce the failure probability estimate.

As we take actions to modify the structure, learn about the random variables, or improve the models, the distribution of $\mathbf{V}$ changes. We show this by changing the superscript $\{0\}$. Specifically, $f_{\mathbf{V}}^{\{a_{1:i}\}}(\mathbf{v})$ denotes the distribution of $\mathbf{V}$ after an ordered set of actions $\{a_{1:i}\} = \{a_1, \ldots, a_i\}$. The corresponding failure probability estimate is denoted $p_{\mathcal{F}}^{\{a_{1:i}\}}$. Our aim is to find an optimal sequence of future actions $\mathcal{A}_{\text{opt}} = \{a_1, \ldots, a_n\}$ that minimizes the expected costs, while assuring that $p_{\mathcal{F}}^{\{a_{1:i}\}} \leqslant p_{\mathcal{F}}^{\{\text{adm.}\}}$.

3. Optimization framework

This section presents the formulation of the optimization framework for identifying the sequence of future actions that minimizes the expected costs and satisfies the failure probability constraint. Sub-Section 3.1 presents the mathematical formulation of the optimization problem, Sub-Section 3.2 discusses computational issues, and Sub-Section 3.3 describes the effects of structural intervention and information gathering actions on the random variables involved in the estimation of the failure probability.

3.1. Formulation of the optimization framework

As mentioned in Section 2, when $p_{\mathcal{F}}^{\{0\}} > p_{\mathcal{F}}^{\{\text{adm.}\}}$, actions are necessary to reduce the failure probability estimate. Let $\mathcal{A} = \{a_1, \ldots, a_i\}$ denote an ordered set of candidate actions so that action a_i can take place only after actions $\{a_1, \ldots, a_{i-1}\}$ have been completed. Example actions include replacement or strengthening of the structure, measurement of component capacities, measurement of variables involved in the capacity or demand models, proof