Structural Safety 52 (2015) 10–19

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Assessing the correlated performance functions of an engineering system via probabilistic analysis

Xing Zheng Wu *

Department of Applied Mathematics, School of Applied Science, University of Science and Technology Beijing, 30 Xueyuan Road, Haidan District, Beijing 100083, PR China

ARTICLE INFO

Article history:
Received 19 April 2013
Received in revised form 14 July 2013
Accepted 22 July 2014
Available online 18 September 2014

Keywords:
Copula
Correlation
Failure mode
Retaining wall
Pipe flow
Pile foundation

ABSTRACT

An important issue regarding the use of probabilistic predictions for complex engineering systems is characterising the dependence structure among its correlated performance functions, which are driven by dependent or independent basic random variables. The interrelationship of these performance functions can be attributed to the same random variables and the cross correlation among the input parameters. An assessment of joint failure probability for an engineering system is proposed, which is associated with the correlated performance functions using a copula-based method by conveying the dependence structure of the performance functions. The method is demonstrated with four simple engineering problems, i.e., (a) bivariate distribution in which two predetermined performance functions are associated with each other; (b) pile bearing capacity in which the performance functions are related with the soil internal friction and the compressive strength of a concrete pile; (c) pipe flow in which the performance function of three pipes in a sewer system is assessed with six independent random variables; and (d) retaining wall in which the failure criteria for defining the performance functions include overturning failure about the toe point, sliding failure along the base, and bearing capacity instability considering uncertain soil properties. The computational efficiency is evaluated using the results based on the conventional bounding methods. The joint failure probability expressed by copulas provides a means to obtain the joint probabilities of multiple failure modes, which pave the way for an objective description of the overall failure probability of a practical engineering problem.

1. Introduction

Interest in addressing the uncertainties in the assessment of an engineering system has long been demonstrated [2,31]. Traditionally, engineers resort to factors of safety to provide confidence when considering the difficulties associated with quantifying the uncertainties [8]. However, the safety factor approach is questionable because it usually does not take into account the underlying variability within each variable and among variables [2,23]. Despite the inherent difficulty in performing a probabilistic analysis, a number of researchers have attempted to introduce probabilistic aspects to address the uncertainty associated with factor-of-safety analysis in the context of risk analysis, as applied to a wide range of engineering systems [28,31].

For a single failure mode or componental analysis, most probabilistic analyses will fall into one of two categories [14,9]: numerical approximate methods or Monte Carlo simulation. The former usually includes the first-order second-moment (FOSM) method, the first-order reliability method (FORM), and the point-estimate method. To solve the problem effectively, these numerical approximation methods make some simplifying assumptions regarding the performance function. The Monte Carlo method relies on the random sampling of variables from probability distributions, hence an accurate determination of the tail distribution can require excessive computational effort [18,24]. Despite the fact that the necessary sample size to obtain the failure probability with a desired accuracy is independent on the number of input random variables, some of the sampling techniques can have some issues with a large number of random variables [32]. Nevertheless, the Monte Carlo simulation approach is flexible when dealing with a nonlinear limit state function that is explicit in terms of the design variables.

An accurate evaluation of the failure probability in an engineering system is significantly more difficult and subjective than the above component level probability analysis. For multiple failure modes, component failure events are usually dependent: if one component failure mode occurs, then the probability of another component failure mode is typically changed. A source of dependent events can arise from the correlation of these random

* Tel.: +86 18611869118; fax: +86 10 82610538.
E-mail address: xingzhengwu@gmail.com

http://dx.doi.org/10.1016/j.strusafe.2014.07.004
0167-4730/

variables in determining the performance functions. Performance functions formulated by the same variables, such as gravity, strength parameters, and loading conditions, may contribute an additional source of dependence.

The failure probability of a series system theoretically involves multidimensional integration, which is usually difficult to evaluate, especially for structures of practical significance. The efficiency of the computational procedures used for estimating system failure probability have resulted in several numerical approximation approaches, including bounding techniques [5,7,2] and direct approximation methods [11,39] and stochastic finite element analysis. Having introduced the simplifying assumptions of complete correlation or independence among failure modes, the bounding method provides the upper and lower failure probability bounds with very little numerical effort. This is accomplished by incorporating the component failure probability for fully dependent and independent failure events of a complex engineering system. This method has attracted considerable attention recently [31], and in some cases, the approach does not require explicit knowledge of the dependence between failure modes. Simple bounds (including the uni-modal method and bi-modal method, see [2]) are obtained by assuming two extreme values for the correlation between component failures and are usually too wide to be useful for design decisions. To obtain better estimates of the probability of failure in the risk analysis context, the statistical dependence characteristics of the failure events must be measured accurately.

These dependence problems can be solved using a very flexible joint distribution, a copula, which has been well known for some time within the statistical literature. The theory was first mentioned by Sklar [27] and permits independence of the marginal parameters. Any form of marginal distribution can be knitted together to obtain their joint distribution, which is the main reason for the popularity of copula theory in many areas of research [10,16,38]. The re-construction of a joint cumulative distribution function for an engineering system requires the use of a marginal cumulative distribution function (CDF) of the component failure mode and a dependence structure between them; therefore, an efficient numerical approximation approach for estimating the system failure probability can be developed, paving the way to precise joint modelling of multiple failure modes.

The objective of this paper is to establish the joint probability distribution of performance functions through copulas. Using this joint distribution, a fairly accurate estimate of the probability of failure for an engineering system is obtained, and the computational efficiency is demonstrated through four example illustrations. The paper is organised as follows: the definitions of the joint failure probability through copulas among performance functions are described in Section 2. Section 3 provides several numerical examples in civil engineering, and the computed failure probabilities of multiple limit states are compared with the results by the conventional bounding methods. The paper closes with a summary of the key findings and the conclusions in Section 4.

2. Modelling the joint behaviour of performance functions

2.1. The failure probability of series structural systems

To simplify the study of system failure probability, two categories of systems are usually involved. In a series system, the failure of one member automatically leads to the failure of the whole system. An example of such a system is a chain, which fails when the weakest link breaks. In a parallel system, all of the members must fail before the system fails. The latter case is not of interest here.

The limit state separates the design space into 'failure' and 'safe' regions of the system by defining the performance function G. A performance function or limit state function is formulated to describe the corresponding failure mode in terms of a vector of mutually independent or dependent basic random variables $\boldsymbol{Z} = (Z_1, Z_2, ..., Z_n)$ in which Z_i are basic random variables, such as the physical properties of the materials and loading conditions. The measurement of these variables by laboratory tests or observations reveals large variations and inherent uncertainties [21,8]. However, due to the existence of many uncertainties of input variables $\boldsymbol{Z}$, the outputs of performance function G cannot be predicted with certainty; rather, random draws must be used. Additionally, different failure modes will depend on the same random variables, or the variables that contribute to the different failure modes will be correlated in some way so that the failure modes become correlated. A Venn diagram for three failure modes of a series system is shown in Fig. 1 for illustration purposes. When the three failure modes are mutually exclusive, there are no overlaps between any of the sets (depicted in Fig. 1a). When none of the three failure modes is mutually exclusive, there is an overlap among the sets of failure (depicted in Fig. 1b). Hence, the joint failure probabilities, when considering the intersection of these failure modes, should be computed carefully by complex distribution transformations and/or numerical integration.

2.2. Joint distribution of the performance functions using copulas

Assessing the probability distribution function of an engineering system requires a model for the joint behaviours of the random outputs of performance functions for $\boldsymbol{G}$, which include $G_1(\boldsymbol{Z})$, ..., $G_s(\boldsymbol{Z})$ and are abbreviated as $G_1, \ldots, G_s$. Here, s is the number of random outputs of the performance functions whose behaviours are of interest for understanding. The probability distribution function of the component failure mode is written by $P_{\mathrm{f}i}(G_i(\boldsymbol{Z}) \leqslant 0)$, for $i = 1, \ldots, s$. It is clear that a definition of the joint failure distribution for $\boldsymbol{G}$ requires full information concerning the marginal probability functions and the dependence between all of the performance functions. Without considering the interactions between the performance functions, the joint failure probability distribution of $\boldsymbol{G}$ is simply $\prod_{i=1}^{s} P_{\mathrm{f}i}(G_i \leqslant g_i)$. When considering the interactions among the performance functions, the joint failure probability distribution is denoted by $C(f_1(g_1), ..., f_s(g_s))$ or $P_{\mathrm{f}1,...,s}$, here, $f_i(g_i) = P_{\mathrm{f}i}(G_i < g_i)$, i.e., the continuous random variable G_i is transformed to a uniform random variable over the unit interval by its probability integral transformation. Thus, $C(f_1(g_1), ..., f_s(g_s))$ is defined from $[0, 1]^s \to [0, 1]$ and it can be obtained from the dependence structure between performance functions. Let the transformations $f_1(g_1), ..., f_s(g_s)$ be s uniform random variables over the unit interval with a joint multivariate distribution

$$C(f_1(g_1), \ldots, f_s(g_s)) = P(F_1(G_1) \leqslant f_1(g_1), \ldots, F_s(G_s) \leqslant f_s(g_s)) \quad (1)$$

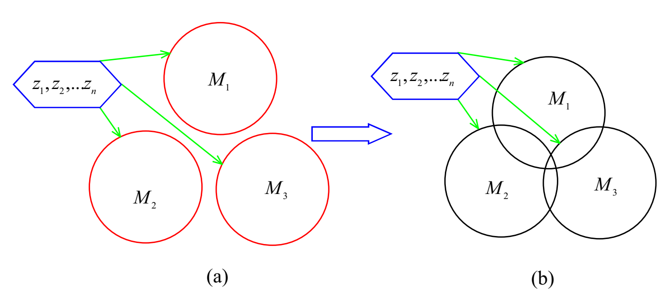

Fig. 1. The Venn diagram for the cases of (a) mutually exclusive failure modes and (b) when none of the three failure modes is mutually exclusive.