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Transient response of stochastic finite element systems using Dynamic Variability Response Functions

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ABSTRACT

In this study a methodology is presented for effective analysis of dynamic systems with stochastic material properties. The concept of dynamic mean and variability response functions, recently established for linear stochastic single degree of freedom oscillators, is extended to general finite element systems such as statically indeterminate beam/frame structures and plane stress problems, leading to closed form integral expressions for their dynamic mean and variability response. The integrand of these integral expressions involves the spectral density function of the uncertain material properties and the so called dynamic mean and variability response functions respectively, which are assumed to be deterministic, i.e. independent of the power spectrum as well as the marginal *pdf* of the uncertain parameters. A finite element method-based fast Monte Carlo simulation procedure is used for the accurate and efficient numerical evaluation of these functions. In order to demonstrate the validity of the proposed procedure, the results obtained using the aforementioned integral expressions are compared to brute-force Monte Carlo simulation. As a further validation of the assumption of independence of the variability response function to the stochastic parameters of the problem, the concept of the generalized variability response function was applied and compared to the steady state dynamic variability response function. The methodology is applied in a dynamically loaded statically indeterminate beam/frame structure and a plane stress problem. The dynamic mean and variability response functions, once established, can be used to perform sensitivity/parametric analyses with respect to various probabilistic characteristics involved in the problem (i.e., correlation distance, standard deviation) and to establish realizable upper bounds on the dynamic mean and variance of the response, at practically no additional computational cost.

1. Introduction

In recent years, multiple methodologies based on perturbation/expansion [1,2], spectral Galerkin approximations [3] or costly Monte Carlo methods [1,4–6] have been developed to deal with random/uncertain phenomena in steady state stochastic structural analysis and extended to dynamic stochastic analysis in a straightforward manner [7,8], along with procedures to improve their efficiency both in terms of accuracy [9–12] as well as computational performance [13–15]. A probability density evolution method was proposed in [16,17] in an effort to approximate the time varying probability distribution function (*pdf*) of the response of stochastic systems using the principle of preservation of probability. Along these lines, some other approaches implement approximate Wiener path integral solution schemes [18]. However these approaches have been mainly implemented in single degree of freedom oscillators or small illustrative academic systems due to increased computational cost. In all above cases, prior knowledge of the correlation properties and the marginal *pdf* of the random fields characterizing system uncertainties is essential for accurate estimates of the system's response. In the frequent case of insufficient experimental data, analysts are forced to resort to sensitivity/parametric yet cost inefficient analyses. Furthermore, such analyses do not provide any information on the mechanisms that affect response variability, or bounds of the response. In addition to the aforementioned approaches, a relatively small number of studies have dealt with the dynamic propagation of system uncertainties, most of them reducing the stochastic dynamic PDE's to a linear random eigenvalue problem [19,20].

In order to effectively resolve aforementioned issues, a proposition has been made through the concept of Dynamic Variability Response Function (*DVRF*) in [21], which was a straightforward generalization of the currently classical *VRF* proposed in the late 1980s [22] along with different aspects and extensions [23,24].

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DVRF involves information regarding deterministic variables of the problem and the standard deviation of the field modeling the random system parameters. In that work, closed form integral expressions involving *DVRF* and the spectral density function of the stochastic field, were suggested for the computation of the dynamic variance of the response displacement as follows:

$$Var[u(t)] = \int_{-\infty}^{\infty} DVRF(t, \kappa, \sigma_{ff}) S_{ff}(\kappa) d\kappa \tag{1}$$

An additional expression involving a Dynamic Mean Response Function (*DMRF*) for the system dynamic mean response was also proposed in that work. This approach was formulated for linear statically determinate single degree of freedom stochastic oscillators under dynamic excitations where it was demonstrated that the integral form expressions for the dynamic mean and variance can be used to effectively compute the first and second order statistics of the transient system response with reasonable accuracy, together with time dependent spectral-distribution-free upper bounds. They also provide an insight into the mechanisms controlling the uncertainty propagation with respect to both space and time and in particular the mean and variability time histories of the stochastic system dynamic response. Furthermore, once the *DMRF* and *DVRF* are established, sensitivity analyses with respect to various probabilistic parameters such as correlation distances and standard deviation were performed at a very small additional computational cost.

Based on the aforementioned recent development, closed form integral expressions in the form of Eq. (1) are proposed in the present work for the mean and variance of the dynamic response of statically indeterminate beam/frame structures and then extended to more general stochastic finite element systems (i.e. plane stress problems) under dynamic excitations. In this case **DVRF** and **DMRF** are vectors comprised of a *DMRF* and *DVRF* for each degree of freedom of the FE system. A general so-called Dynamic FEM fast Monte Carlo simulation (DFEM-FMCS) is provided for the accurate and efficient evaluation of **DVRF** and **DMRF** for stochastic FE systems. Numerical results are presented, demonstrating that, as in the case of classical *VRFs*, as well as in the case of *DMRF* and *DVRF* for single degree of freedom stochastic oscillators [21], the **DVRF** and **DMRF** matrices appear to be independent of the functional form of the power spectral density function $S_{ff}(\kappa)$ and appear to be marginally dependent on the *pdf* of the field modeling the uncertain system parameter. It is reminded that the existence of *VRF* has been proven only in the case of statically determinate structures under static loading [22,25]. In all other cases this existence had to be conjectured and the validity of this conjecture was demonstrated through comparisons of the results obtained from Eq. (1) with brute force MCS. The validity of this conjecture is further boosted in this work by comparing steady state *DVRF* with respective *Generalized VRF* [26] for a statically indeterminate frame structure. *GVRF* involves the computation of different *VRFs* for corresponding combinations of different marginal *pdfs* and power spectra nd was developed in order to further test the validity of the existence of a *VRF* which is almost independent of the stochastic parameters of the problem. It should be mentioned here that the *VRF* concept was recently extended in [29] for structures with non-linear material properties where a closed form analytic expression of *VRF* revealed the clear dependence of the integral form of Eq. (1) on the standard deviation as well as higher order Power spectra of $f(x)$. Finally, realizable upper bounds of the mean and dynamic system response are evaluated.

2. Time-history analysis of stochastic finite element systems

Without loss of generality consider the linear stochastic FE system of Fig. 1 which is a fixed–fixed beam/frame structure. The inverse of the elastic modulus is assumed to vary randomly along its length according to the following expression:

$$\frac{1}{E(x)} = F_0(1 + f(x)), \tag{2}$$

where E is the elastic modulus, F_0 is the mean value of the inverse of E, and $f(x)$ is a zero-mean homogeneous stochastic field modeling the variation of $1/E$ around its mean value.

For the derivation of the deterministic system dynamic response the trivial second-order differential equation for the discretized FE dynamic system equilibrium is as follows:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{P}(t) \tag{3}$$

where $\mathbf{M}$ is the mass matrix of the discretized FE system, $\mathbf{C}$ is its damping matrix, $\mathbf{K}$ is its stiffness matrix and $\mathbf{P}(t)$ is its loading vector. At last, $\mathbf{u}(t)$ is the time-history of the displacement vector of the system, providing information about the response of each node of the FE mesh, $\dot{\mathbf{u}}(t)$ is the first order time-derivative and $\ddot{\mathbf{u}}(t)$ is the second order time-derivative of $\mathbf{u}(t)$.

Direct integration of Eq. (3) can be performed using i.e. a Newmark scheme of the following form:

$${}^{t+\Delta t}\hat{\mathbf{R}} = {}^{t+\Delta t}\mathbf{R} + \mathbf{M}(a_0{}^t\mathbf{U} + a_1{}^t\dot{\mathbf{U}} + a_2{}^t\ddot{\mathbf{U}}) + \mathbf{C}(a_1{}^t\mathbf{U} + a_4{}^t\dot{\mathbf{U}} + a_5{}^t\ddot{\mathbf{U}}) \tag{4}$$

where $a_0 = \frac{1}{a\Delta t^2}; a_1 = \frac{1}{a\Delta t}; a_2 = \frac{1}{2a} - 1; a_4 = \Delta t(1-\delta); a_5 = \delta\Delta t; a_6 = \Delta t(1-\delta); a_7 = \Delta t$. After choosing a time step Δt parameters α and δ are selected under the limitations $\delta \geqslant 0.50$ and $a \geqslant 0.25(0.5+\delta)^2$. After initialization of ${}^0\mathbf{U}, {}^0\dot{\mathbf{U}}$, and ${}^0\ddot{\mathbf{U}}$, the displacements at time $t+\Delta t$ are calculated solving the following linear system of equations

$$\hat{\mathbf{K}}{}^{t+\Delta t}\mathbf{U} = {}^{t+\Delta t}\hat{\mathbf{R}} \tag{5}$$

where $\hat{\mathbf{K}}$ is the effective stiffness matrix given by

$$\hat{\mathbf{K}} = \mathbf{K} + a_0\mathbf{M} + a_1\mathbf{C} \tag{6}$$

Finally accelerations and velocities at time $t+\Delta t$ accrue from the following equations:

$${}^{t+\Delta t}\ddot{\mathbf{U}} = a_0({}^{t+\Delta t}\mathbf{U} - {}^t\mathbf{U}) - a_1^t\dot{\mathbf{U}} - a_2^t\ddot{\mathbf{U}} \tag{7}$$

$${}^{t+\Delta t}\dot{\mathbf{U}} = {}^t\dot{\mathbf{U}} + a_6{}^t\ddot{\mathbf{U}} + a_7^{t+\Delta t}\ddot{\mathbf{U}} \tag{8}$$

Matrices $\hat{\mathbf{R}}$ and $\hat{\mathbf{K}}$ in Eqs. (5) and (6) and consequently vectors $\mathbf{U}, \dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are random due to the variation of $E(x)$ in Eq. (2). Thus, the solution of Eq. (5) requires the implementation of some stochastic methodology in order to invert the stochastic operator $\hat{\mathbf{K}}$ in at each time step and predict the stochastic dynamic response of the FE system.

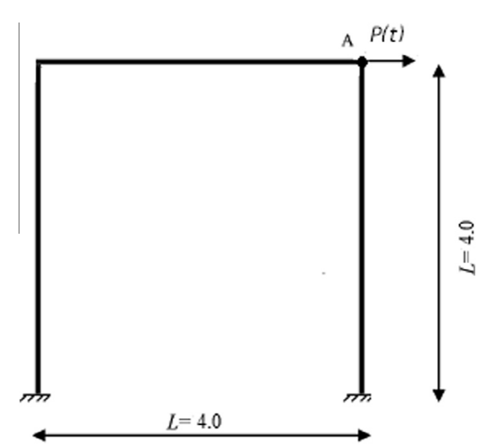

Fig. 1. Geometry and loading of the fixed–fixed frame discretized with 60 beam elements.