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Impact of spatial variability in undrained shear strength on active lateral force in clay

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ABSTRACT

This study explores the mechanism of the active lateral force (P_a) in clay when there is spatial variability in the undrained shear strength. It is shown that the effect of such spatial variability cannot be fully explained by considering the spatial averaging over a prescribed area or line only; the mechanism to seek the favorable failure path is also important. Ignoring this important mechanism is risky, rendering the P_a estimate smaller than the actual P_a value. The mechanism of seeking the favorable failure path is explored, and a probability distribution model for P_a is proposed to characterize this sophisticated mechanism. Furthermore, a simplified procedure is proposed to simulate the P_a samples without the use of the finite element method or limit equilibrium method.

1. Introduction

Active lateral forces of soils are commonly calculated by traditional earth pressure theories, such as the Rankine and Coulomb theories. These theories were developed based on the assumption that the soil is homogeneous. However, soils are spatially variable due to natural geologic formation processes. The impact of soil spatial variability on lateral forces has been studied by Fenton et al. [5] for active lateral force (P_a) and by Griffiths et al. [8] for passive lateral force (P_p). In these studies, the spatial variability of the friction angle for the backfill sand is modeled as a stationary random field, and lateral forces are simulated by random field finite elements. The failure probability (p_f) of the retaining wall is found to be closely related to the coefficient of variation (COV) and the scale of fluctuation (SOF) of the random field. More interestingly, a critical scale of fluctuation is observed when p_f becomes significantly larger. For the case of P_a, the critical SOF is found to be close to the height of the retaining wall.

The main purpose of this study is to explore the mechanism of the active lateral force in clay in the presence of spatial variability in the undrained shear strength (s_u). In particular, the following question will be addressed: can the effect of the s_u spatial variability be summarized as the spatial average over a prescribed area or line, e.g., averaging over the area above the classical slip line with an inclination angle of 45° or averaging along this classical slip line? If the spatial averaging alone cannot fully characterize the effect of s_u spatial variability, what are the missing factors and how to quantify those factors?

For a clay specimen subjected to a uniform stress state (compression or shear), Ching and Phoon [2] have shown that the effect of the s_u spatial variability cannot be summarized as the spatial average over a prescribed area or line. Moreover, Ching and Phoon [3] and Ching et al. [4] concluded that two factors govern the behaviors of the mobilized shear strength (mobilized s_u) for such a clay specimen:

(a) The averaging effect along the potential slip line (PSL). This factor was quantified by the variance reduction factor due to the line averaging effect along the PSL.
(b) The emergent feature of the critical slip line, i.e., the phenomenon that the mobilized shear strength depends on the trajectory of the critical slip line passing through the spatially varying soil mass and this trajectory is unknown a priori – the critical slip line may seek the favorable path. This emergent feature was further quantified by the "number of independent slip lines" in Ching and Phoon [3].

Based on these two factors, they further proposed equations and models to predict the mean, variance, and the probability distribution of the mobilized s_u for such a clay specimen. They showed that the phenomenon of the critical scale of fluctuation can happen when these two factors interact with each other.

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Nomenclature

ρ	auto-correlation	LEM	limit equilibrium method
Φ	CDF of the standard normal random variable	L_p	length of the potential slip line
ϕ	friction angle	L_x	horizontal dimension of the simulation domain
β	inclination angle of the potential slip line	L_z	vertical dimension of the simulation domain
μ	inherent mean of the $\tau_f(x,z)$ random field	n	number of independent potential slip lines
σ	inherent standard deviation of the $\tau_f(x,z)$ random field	N	sample size
φ	PDF of the standard normal random variable	P_a	active lateral force
γ	unit weight	$P_a^{AA}(45°)$	P_a value based on $\tau_f^{AA}(45°)$
β^*	inclination angle of the critical slip line	$P_a^{LA}(45°)$	P_a value based on $\tau_f^{LA}(45°)$
Γ^2	variance reduction factor	PDF	probability density function
τ_f	shear strength	p_f	failure probability
$\tau_f(x,z)$	the random field for τ_f	P_p	passive lateral force
τ_f^{AA}	area average of $\tau_f(x,z)$ over the area above a potential slip line	PSL	potential slip line
τ_f^{LA}	line average of $\tau_f(x,z)$ along a potential slip line	RFEM	random field finite element method
τ_f^m	mobilized shear strength	SOF	scale of fluctuation
δ_p	SOF along the potential slip line	s_u	undrained shear strength
δ_x	horizontal scale of fluctuation	W	weight of the wedge
δ_z	vertical scale of fluctuation	x	horizontal coordinate
COV	coefficient of variation	z	vertical coordinate
F	lateral force	z_c	depth of vertical tension crack
FEA	finite element analysis	Δx	horizontal distance
H	wall height	Δz	vertical distance

However, for a retaining wall problem, the slip line is partially constrained because it must pass through the toe (see Fig. 1a). It is expected that the above factor (b) (emergent feature of the critical slip line) should be less significant. An important goal of this study is to verify whether the effect of the s_u spatial variability can still be summarized by the above two factors? Maybe factor (a) alone is sufficient because factor (b) is less influential? Finally, a model will be developed to predict the probability distribution of the mobilized s_u for a retaining wall in clay. With this model, the active lateral force P_a can be simulated without conducting the more complicated random field finite element method (RFEM).

2. Simulation of active lateral forces

This section presents the use of the limit equilibrium method (LEM) for simulating P_a when there is spatial variability in clay. The undrained shear strength at a point is denoted by $\tau_f(x,z)$, where x and z are the horizontal and vertical coordinates, respectively. The friction angle ϕ is taken to be 0°, i.e., an undrained clay is considered. Therefore, the shear strength (τ_f) = the undrained shear strength (s_u). The shear strength $\tau_f(x,z)$ is simulated as a stationary Gaussian random field with inherent mean $E(\tau_f) = \mu$ and inherent standard deviation $[\mathrm{Var}(\tau_f)]^{0.5} = \sigma$. The COV of this random field is equal to σ/μ. The spatial variability in the clay shear strength is the only uncertainty considered in this study: we do not consider other uncertainties such as modeling uncertainties and measurement errors.

To define the correlation structure of $\tau_f(x,z)$ between two locations with horizontal distance = Δx and vertical distance = Δz, the single exponential (SExp) auto-correlation model is considered [12,13]:

$$\rho(\Delta x, \Delta z) = \exp\left(-\frac{2|\Delta x|}{\delta_x} - \frac{2|\Delta z|}{\delta_z}\right) \quad (1)$$

where δ_x and δ_z are the horizontal and vertical scales of fluctuation (SOFs), respectively, that describe the distance within which the properties of the soil exhibit considerable correlation. For simplicity, only the cases with the case with $\delta_x = \delta_z = \delta$ is considered in this study.

A process called "line search" for simulating a random sample of P_a using the LEM is illustrated in Fig. 1. Fig. 1a shows a realization of the $\tau_f(x,z)$ random field, β_A, β_B, and β_C in Fig. 1a characterize the inclination angles of the three potential slip lines (PSLs). It is assumed that the PSLs are straight lines. This assumption is in principle unconservative because the critical failure path may not be a straight line. However, in a later comparison with the RFEM results (note: RFEM does not assume a slip "line"), it will be shown that the P_a value simulated by the LEM is fairly consistent to that simulated by the RFEM. Hence, the actual effect of this "straight line" assumption is minimal. It is also assumed that the PSLs pass through the toe. This is barely an assumption because the critical slip lines obtained in the RFEM always pass through the toe as well.

There are actually infinite PSLs, but for clarity, only three of them are shown. Each PSL produces a "line average", denoted by τ_f^{LA}. This line average is simply the spatial average of the $\tau_f(x,z)$ random field along each PSL. This line average τ_f^{LA} can be directly simulated by the Fourier series method developed in Jha and Ching [9] (Eq. (19) in Jha and Ching [9]):

$$\tau_f^{LA} = \frac{L_x L_z}{2\pi} Re\left\{\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{a_{mn}+ib_{mn}}{im\Delta x L_z + in\Delta z L_x}\exp\left(\frac{i2m\pi x_0}{L_x}+\frac{i2n\pi z_0}{L_z}\right) \times \left[exp\left(\frac{i2m\pi\Delta x}{L_x}+\frac{i2n\pi\Delta z}{L_z}\right)-1\right]\right\} \quad (2)$$

where Re[.] refers to the real part of the enclosed complex number; L_x and L_z are the horizontal and vertical dimensions of the simulation domain (L_x = 17.5 m and L_z = 5 m for the special case in Fig. 1a), respectively; a_{mn} and b_{mn} are independent zero-mean Gaussian random variables with the following variance:

$$\sigma_{mn}^2 = \frac{\sigma^2\cdot\delta_x\delta_z}{L_x L_z}\left[\frac{1-\exp(-L_x/\delta_x)\cdot(-1)^m}{1+m^2\pi^2\delta_x^2/L_x^2}\right]\left[\frac{1-\exp(-L_z/\delta_z)\cdot(-1)^n}{1+n^2\pi^2\delta_z^2/L_z^2}\right] \quad (3)$$

(x_0, z_0) and (x_1, z_1) are the coordinates of the two end points of the PSL; $\Delta x = x_1 - x_0$; and $\Delta z = z_1 - z_0$. For the coordinate system shown in Fig. 1a, the left end point has coordinates $x_0 = z_0 = 0$; thus, $\Delta x = x_1$