Structural Safety 51 (2014) 29–34

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

A sparse grid stochastic collocation method for structural reliability analysis

Jun He *, Shengbin Gao, Jinghai Gong

Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

ARTICLE INFO

Article history:
Received 22 April 2013
Received in revised form 28 May 2014
Accepted 2 June 2014
Available online 2 July 2014

Keywords:
Smolyak-type quadrature formula
Moment-based Gauss transformation
Probability moment
Random parameter and load
Structural reliability

ABSTRACT

This paper develops a sparse grid stochastic collocation method for the reliability analysis of structures with uncertain parameters and loads. The method consists of two standard techniques in uncertainty quantification: the moment-based Gauss transformation and Smolyak-type sparse grid quadrature rule. Unlike the first-order reliability method (FORM) or second-order reliability method (SORM), the developed method does not need the evaluation of the first- or second-order partial derivatives of the limit state function considered and, moreover, does not suffer from the problem of multiple design points. In addition, the developed method is suitable for all problems whose deterministic solutions can be found and usually needs much fewer function evaluations than the Monte Carlo simulation method. Numerical examples demonstrate that the developed method is exact enough for evaluating the mean values, standard deviations, skewness and kurtosis of the limit state functions and small probabilities of failure as low as 10^{-4}. Even for probabilities of failure as low as 10^{-5}, the quality of approximation obtained by the method is also acceptable.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

The fundamental problem in the structural reliability analysis is to determine the probability of failure, i.e. the probability that the total load effect (total demand), *Q*, will exceed in magnitude the resistance (or capacity), *R*. In practical applications, the resistance and the total load effect are usually implicit or nonlinear functions of a set of random variables modeling external loads or structural parameters, $Y = \{Y_i\}_{i=1}^N$, it is often impossible, therefore, to exactly determine the cumulative distribution functions (CDFs) or probability density functions (PDFs) of *R*, *Q*, and the limit state function, $Z = R - Q$. As a sequence, the exact determination of the probability of failure corresponding to the limit state function is impossible.

Various approximate methods for the determination of the probability of failure have been developed in the past several decades. Among them, the first-order reliability method (FORM) is the most widely-used method which can provide sufficiently accurate solution in the slightly nonlinear cases [1,2]. To improve the accuracy of FORM for the strongly nonlinear cases, the second-order reliability method (SORM) has been developed [3–5]. However, both FORM and SORM are nontrivial in the sense that they need the determination of the first- and second-order partial derivatives of the limit state function, $Z(Y)$, with respect to the input random variables, $Y = \{Y_i\}_{i=1}^N$. On the other hand, they may also encounter the problem of multiple design points in practical applications [6].

An alternative numerical method for the determination of the probability of failure is the Monte Carlo simulation method, which generates ensembles of random realizations for the input random variables and utilizes repetitive deterministic solvers for each realization [2,7]. This method is suitable for all problems whose deterministic solutions can be found, while it may need infeasible computational effort to estimate the low probability of failure. To save a part of the computational effort, the directional simulation method [6], Latin Hypercube sampling method [2,8], quasi-Monte Carlo method [9] and Markov chain Monte Carlo method [10] have been developed. However, additional restrictions are imposed on them and their application is limited.

In the last several years, the stochastic collocation method has been developed for uncertainty quantification [11,12]. The stochastic collocation method is based on two techniques in numerical analysis: functional interpolation via Lagrange polynomials and integration of functions via quadrature rules. In the structural reliability analysis, the former can be used to construct a polynomial approximation for the implicit or nonlinear limit state function, $Z(Y)$, and the latter can be used to obtain approximate estimates of the first *n*th moments of $Z(Y)$, such as the mean value μ_Z, standard deviation σ_Z, skewness $\alpha_{3,Z}$, kurtosis $\alpha_{4,Z}$ and so on. Based

* Corresponding author. Tel.: +86 21 3420 6697; fax: +86 21 3420 6698.
E-mail address: junhe@sjtu.edu.cn (J. He).

http://dx.doi.org/10.1016/j.strusafe.2014.06.003
0167-4730/© 2014 Elsevier Ltd. All rights reserved.

on the polynomial approximation or first nth moments, one can approximately determine the probability of failure through using the Monte Carlo simulation method or moment-based Gauss transformation [13–15].

The core issue for the stochastic collocation method is the construction of the set of interpolation points in the multidimensional space of the input random variables. There exist two commonly-used methods for the construction, i.e. the tensor product method [12,16] and sparse grid method [17–19]. The sparse grid method can generate much fewer interpolation points and, therefore, is a more efficient tool to construct the set of interpolation points [20]. Actually, the point estimate method [21] or decomposition method [22] for the structural reliability analysis developed recently can be regarded as a simplified version of the sparse grid stochastic collocation method, as described in Appendix B.

The principle objective of the present study is to develop a sparse grid stochastic collocation method for the structural reliability analysis. The main idea of the method is to approximately determine the probability of failure through using the first 4th moments of the limit state function, which are computed by the Smolyak-type quadrature formula. The use, computational accuracy and numerical efficiency of the developed method are demonstrated by reliability analyses of a steel beam with an explicitly nonlinear limit state function and a steel frame structure whose limit state function is implicit and constructed by using the finite element analysis.

2. Probability moments and failure probability

Without loss of generality, consider the following limit state function for the structural reliability analysis

$$Z(Y) = R(Y) - Q(Y) \tag{1}$$

in which the resistance, $R(Y)$, and total load effect, $Q(Y)$, are nonlinear or implicit functions with respect to the input random variables, $Y = \{Y_i\}_{i=1}^N$, modeling the external loads and structural parameters.

Mathematically, the probability of failure corresponding to the limit state function is

$$P_f = P(Z \leqslant 0) \tag{2}$$

where $P(\cdot)$ denotes the probability of an event.

As already mentioned, it is often impossible to exactly determine the probability of failure since CDFs or PDFs of R, Q, and Z are usually unknown. However, one can approximately estimate the probability of failure from the first 4th moments of Z through using moment-based Gauss transformations. The first 4th moments of Z, i.e. the mean value μ_Z, standard deviation σ_Z, skewness $\alpha_{3,Z}$ and kurtosis $\alpha_{4,Z}$, are defined as

$$\mu_Z = \int_{D_{\mathbf{Y}}} Z(\mathbf{y}) f_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y} \tag{3}$$

$$\sigma_Z^2 = \int_{D_{\mathbf{Y}}} [Z(\mathbf{y}) - \mu_Z]^2 f_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y} \tag{4}$$

$$\alpha_{3,Z} = \frac{1}{\sigma_Z^3} \int_{D_{\mathbf{Y}}} [Z(\mathbf{y}) - \mu_Z]^3 f_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y} \tag{5}$$

$$\alpha_{4,Z} = \frac{1}{\sigma_Z^4} \int_{D_{\mathbf{Y}}} [Z(\mathbf{y}) - \mu_Z]^4 f_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y} \tag{6}$$

in which D_Y is the domain of $\mathbf{Y}$, $Z(\mathbf{y})$ is the solution of $Z(\mathbf{Y})$ obtained from Eq. (1) by replacing $\mathbf{Y}$ by deterministic values $\mathbf{y}$, and $f_Y(y)$ is the joint PDF of $\mathbf{Y}$.

The probability of failure based on the first 4th moments can be approximately obtained from

$$P_f = \Phi[T(0)] \tag{7}$$

where $\Phi(\cdot)$ denotes the CDF of a standard normal random variable, and $T(\cdot)$ denotes a moment-based Gauss transformation, e.g. the second order Hermite model as described in Appendix A.

From Eq. (7), one can obtain the generalized reliability index

$$\beta = -T(0) \tag{8}$$

It can be seen in Eqs. (3)–(6) that to find the first 4th moments of the limit state function one only needs to compute the multivariate integration

$$I^{(n)} = \int_{D_{\mathbf{Y}}} [Z(\mathbf{y})]^n f_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y}, \;\; n = 1, 2, 3, 4 \tag{9}$$

In general, the multivariate integration must be computed by numerical methods. Since the evaluation of $Z(\mathbf{y})$ is often time-consuming, it is necessary to reduce the number of function evaluations of $Z(\mathbf{y})$ to save the total computational effort.

3. Smolyak-type quadrature formula for multivariate numerical integration

The input random variables, $\mathbf{Y} = \{Y_i\}_{i=1}^N$, may be mutually dependent in practical applications, while it is assumed here that they are mutually independent. The assumption of independence is not a loss of generality, since it is always possible to transform the mutually dependent random variables to mutually independent random variables [2,6,23]. Besides, the Rosenblatt transformation [6,23], $\mathbf{Y}_i = G_i(X_i)$, is also employed so that the input random variables, $\mathbf{Y} = \{Y_i\}_{i=1}^N$, are expressed by a set of mutually independent standard normal random variables, $\mathbf{X} = \{X_i\}_{i=1}^N$. Thus, the multivariate integration, Eq. (9), can be rewritten as

$$I^{(n)} = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \{Z[G_1(x_1), \ldots, G_N(x_N)]\}^n \varphi(x_1) \ldots \varphi(x_N) dx_1 \ldots dx_N \tag{10}$$

where $\varphi(\cdot)$ is the PDF of a standard normal random variable. The joint PDF of $X = \{X_i\}_{i=1}^N$ in Eq. (10) can be regarded as the weight associated with the kernel $\{Z[G_1(x_1), \ldots, G_N(x_N)]\}^n$.

From the Smolyak algorithm [17], the sparse grid quadrature formula for the numerical computation of the multivariate integration Eq. (9) can be derived as, see Appendix B

$$\begin{aligned}\widehat{I}(n) = &\sum_{\mathbf{i} \in H(q,N)} (-1)^{q+N-|\mathbf{i}|} \binom{N-1}{q+N-|\mathbf{i}|} \\ &\times \sum_{j_1=1}^{m_{i_1}} \cdots \sum_{j_N=1}^{m_{i_N}} \{Z[G_1(x_{j_1}^{i_1}), \ldots, G_N(x_{j_N}^{i_N})]\}^n p_{j_1}^{i_1} \ldots p_{j_N}^{i_N}\end{aligned} \tag{11}$$

where the abscissas and weights are $x_{j_i}^{i_i} = \sqrt{2}\xi_{j_i}^{i_i}$ and $p_{j_i}^{i_i} = \frac{1}{\sqrt{\pi}}\zeta_{j_i}^{i_i}$, $j_i = 1, \ldots, m_{i_i}$, in which $\xi_{j_i}^{i_i}$ and $\zeta_{j_i}^{i_i}$ are the abscissas and weights in the Gauss–Hermite quadrature formula, the multi-index $\mathbf{i} = (i_1, \ldots, i_N) \in \mathbb{N}_+^N$, the nonnegative integer q is the *exactness level*, and the set $H(q, N)$ is defined by

$$H(q, N) = \{\mathbf{i} \in \mathbb{N}_+^N, \mathbf{i} \geq 1 : q + 1 \leq \sum_{n=1}^N i_n \leq q + N\} \tag{12}$$

in which $\mathbf{1} = (1, \ldots, 1)$.

The choice of q and the definition of m_i depends on the nonlinearity of the limit state function considered and are discussed in Appendix B.

4. Errors in the method

Since the developed reliability method consists of the moment-based Gauss transformation and Smolyak-type sparse grid quadrature rule, the overall error of the method can be roughly

Download English Version:

https://daneshyari.com/en/article/307529

Download Persian Version:

https://daneshyari.com/article/307529

Daneshyari.com