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A novel reliability method for structural systems with truncated random variables

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ABSTRACT

Uncertainty is usually modeled using random variable with certain probability distribution. However, the probability distributions of many random variables are often truncated in engineering applications. In the procedure of reliability based design optimization for structural systems with truncated random variables, repeated function evaluations are required for different design points where the computational costs are extremely huge. In this paper, an efficient as well as novel reliability method is proposed for structural systems with truncated random variables which does not require repeated function evaluations for the different design points. Uniformly distributed samples are generated for truncated random variables in the supported intervals and design variables in the specified intervals to approximate cover the entire uncertain space fully. In order to avoid repeated function evaluations and improve computational efficiency, a surrogate model is established using back-propagation (BP) neural networks which can approximate the relationships between the inputs and system responses properly in almost entire uncertain space using the proposed given available data. The main advantages of the proposed method are high accuracy and effectiveness in estimating the probability of failure under different design points which requires neither large samples nor the repeated function evaluations when compared to the existing reliability methods. Four numerical examples are investigated to demonstrate the effectiveness and accuracy of the proposed method.

1. Introduction

Uncertainty widely exists in engineering practices which can be divided into aleatory and epistemic uncertainties [1–4]. Various uncertainties are usually modeled using random variables. It is well known that the supported intervals of many continuous random variables are $[-\infty, \infty]$ while is impossible in engineering practices. In order to handle the problem, the probability distributions of some random variables are usually truncated. Therefore, truncated variables are involved in many engineering applications. For example, the volatility of material properties and physical dimensions is modeled using truncated random variables in reliability engineering. The truncated exponential distribution is usually employed to model the earthquake magnitude [5,6].

In the reliability-based design optimization (RBDO), repeated reliability estimation is required for each configuration of the design variable. Therefore, reliability analysis is the key step in RBDO. Reported existing reliability analysis methods, such as the first/s order reliability method (FORM/SORM) and Monte Carlo simulation (MCS), can be employed to calculate the probability of failure for structural systems with truncated random variables. However, some studies pointed out that the convergence problem may arise when the standard algorithm of the FORM is employed for structural systems with truncated random variables [7]. To overcome the problem, a modification of the standard FORM algorithm have been proposed by Melchers et al. [7], while the most probable point (MPP) search is also required. Du and Hu [5] linearized limit-state function at the MPP and a reliability method for system with truncated random variable based on the first order saddle-point approximation is presented. They have proved that the accuracy of their proposed method is higher than the FORM while keeps the same efficiency. Despite these efforts, it is well known that the MPP search is an iterative optimization process, which is not only time-consuming for structural systems with implicit performance function, but also needs the repeated function evaluations. Sometimes it may fail when the MPP search process does not converge [8]. MCS can be used for structural systems with truncated random variables; however, the computational costs of the MCS are extremely huge because it requires large sample sizes

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and many repeated function evaluations [8,9]. The computational burden using MCS is extremely huge when the performance function cannot be defined explicitly.

Despite some efforts have been made, RBDO for structural systems with truncated random variables is a challenging problem. Many classical reliability methods (such as the FORM/SORM) can be employed to calculate the probability of failure for structural systems with one failure mode. However, engineering system often has multiple failure modes, and these failure modes are usually correlated each other because they depend on the same uncertain variables. Up to now, the research of system reliability analysis almost had been stagnant when compared to the significant advances of component reliability due to the complicated features and intersections for the multiple failure modes, as well as the many existing methods cannot estimate system probability of failure with high efficiency and accuracy [10,11]. Due to the difficulties, the bounds of system probability of failure are provided by many reported reliability methods, instead of its unique value [12–14]. In RBDO for structural systems with truncated random variables, sometimes it is difficult to use the classical reliability analysis methods (such as the FORM/SORM) due to the MPP search algorithm may completely breakdown [5,7], and the results usually are bounded rather than unique value. Furthermore, repeated function evaluations under different design points are required for many existing methods. Therefore, the computational burden by using these methods is very huge, especially when the finite element analysis (FEA) method is used. In order to avoid the MPP search and repeated function evaluations as well as the difficulties of modeling the multiple failure modes, an efficient reliability method is proposed for structural systems with truncated random variables in this paper. The proposed method is robustness which is suitable for structural systems with explicit or implicit functions and even multiple failure modes. The main advantages of the proposed method are high accuracy and effectiveness in estimating the probability of failure for structural systems under different design points because it does not require the MPP search and large samples, as well as repeated function evaluations.

This paper is organized as follows. Section 2 provides an efficient reliability method for structural systems with truncated distributions in details. Three engineering examples and one mathematical problem are investigated in Section 3 to demonstrate the accuracy and efficiency of the proposed method. A brief discussion and conclusions are provided in Section 4 of the paper.

2. The efficient proposed method for calculating the probability of failure under different design points

The problem of MPP search may breakdown, and the difficulties of modeling multiple failure modes, as well as repeated function evaluations, are main disadvantages of the existing reliability methods for structural systems with truncated random variables. The computational costs for repeated function evaluations are extremely huge, especially for systems with implicit performance functions. In this case, every calculation of system response is a complete finite element calculation which is time costly. To overcome the drawbacks of the existing reliability methods and the difficulties of modeling multiple failure modes as well as improve the accuracy, the combinations of surrogate model and MCS are considered for structural systems, and an efficient reliability method is proposed to calculate the probability of failure for structural systems with truncated random variables.

The following steps are suggested to calculate the probability of failure under different design points for structural systems with truncated random variables in the proposed method.

(1) Generating uniformly distributed samples for truncated random variables in the supported intervals and for design variables in the specified bounded intervals;
(2) Calculating system responses and using the available data to construct a back-propagation (BP) neural network;
(3) Given input samples and calculate the probability of failure under different design points using the trained BP network.

The detailed information for each step is given in the following subsections.

2.1. Generating uniformly distributed samples for truncated random variables in the supported intervals and design variables in the specified bounded intervals

Let $\widetilde{X}_i$ is a continuous random variable with the probability density function (PDF) and cumulative density function (CDF) $f(\widetilde{X}_i)$ and $F_{\widetilde{X}_i}(\widetilde{X}_i)$, respectively, and both of them have infinite supported intervals. Suppose X_i denotes the truncated random variable for $\widetilde{X}_i$ with the supported interval $[a_i, b_i]$, a_i and b_i are two constants, $-\infty < a_i \leqslant x \leqslant b_i < \infty$. The corresponding truncated PDF $f(X_i)$ and CDF $F_{X_i}(X_i)$ of X_i can be respectively given by

$$f(X_i) = \frac{f(\widetilde{X}_i)}{F_{\widetilde{X}_i}(b_i) - F_{\widetilde{X}_i}(a_i)} \tag{1}$$

$$F_{X_i}(x_i) = \frac{1}{F_{\widetilde{X}_i}(b_i) - F_{\widetilde{X}_i}(a_i)}\left[F_{\widetilde{X}_i}(x_i) - F_{\widetilde{X}_i}(a_i)\right] \tag{2}$$

For example, suppose $\widetilde{X}$ is the standard normal distribution, the CDFs for truncated random variable X with supported interval $[-2, 2]$ and $\widetilde{X}$ are shown in Fig. 1.

The generation of pseudo-random numbers is very important and common task in computer programming, which is very useful in developing MCS. The mostly used pseudo-random number generators is the linear congruential generator, which can be expressed as [15]

$$X_{n+1} = (cX_n + d)\text{mod } m \tag{3}$$

where c is called the multiplier, d is called the increment, and m is called the modulus of the generator.

Let u_i denote a uniformly distributed sample in interval $[0, 1]$, the ith uniformly distributed sample in interval $[a, b]$ can be generated by

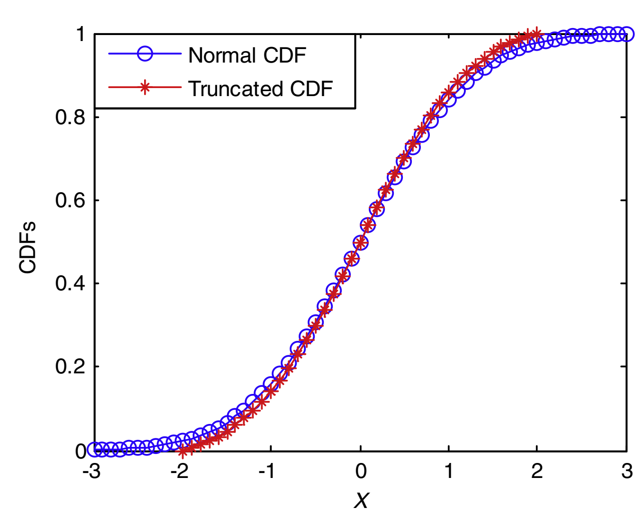

Fig. 1. The CDFs for truncated and standard normal distributions.