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Renewal theory-based life-cycle analysis of deteriorating engineering systems

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ABSTRACT

Engineering systems typically deteriorate due to regular use and exposure to harsh environment. Under such circumstances the owner of a system must take important decisions such as whether to repair, replace or abandon the system. Such decisions can affect the safety of, and the benefits to the users and the owner. Life-cycle analysis (LCA) provides a rational basis for such decision making process. In particular, LCA can provide helpful information on the performance of a system over its entire life-cycle, like its time-dependent reliability, the costs associated with its operation, and other quantities related to the service life of the system.

This paper proposes a novel probabilistic formulation for LCA of deteriorating systems named Renewal Theory-based Life-cycle Analysis (RTLCA). The formulation includes equations to obtain important life-cycle variables such as the expected time lost in repairs, the reliability of the system and the cost of operation and failure. The proposed RTLCA formulation is based on renewal theory and proposes analytical solutions for the desired LCA variables using numerically solvable integral equations. As an illustration, the proposed RTLCA formulation is implemented to analyze the life-cycle of an example reinforced concrete (RC) bridge located in a seismic region. This analysis accounts for the accumulated seismic damage in the bridge columns caused by the earthquakes occurring during bridge's life-cycle. The analysis results provide valuable insight into the importance of seismic damage in a bridge's life-cycle performance and the strategies to operate a system in an optimal manner.

1. Introduction

Engineering systems have to be operated in a strategic manner in order to maximize the safety of, and the benefits to the users and owners. Such operation strategies can be effectively devised only by conducting a life-cycle analysis (LCA) of the system. In LCA, the performance of a system, over its entire life-cycle, is studied in terms of a variety of performance measures such as reliability and its dependence on age of the system, the costs and benefits of operation a system taking into account the influence of repairs and maintenances. LCA must factor in the uncertainties in the operating conditions (e.g., environmental conditions, intensity and time of occurrence of loads) and, for deteriorating systems, it is extremely critical to model the process of deterioration and its effect on life-cycle performance of the system.

In the past, several researchers have developed models for LCA using the well known renewal theory. Renewal theory based models attract attention because they minimize the need for computationally expensive simulations and offer analytical equations to estimate the life-cycle performance measures for a system. However, renewal theory models are built on a well known assumption i.e., when a system is repaired its original properties are restored. Any form of repair is considered to be a complete renewal of the system and therefore partial repairs cannot be handled in renewal theory. Furthermore, all renewal cycles are (i.e. time period between two renewals) are assumed to be independent of each other. With these assumptions, researchers have time to time proposed various LCA models. Rackwitz [1] proposes equations to compute the expected values of benefit derived from operating a system, the cost of failures, the availability of the system and to optimize the design of system based on minimum failure rates. The work develops all the equations for infinite time horizon and for only one type of failure of the system (i.e. either service ability or ultimate failure). Streicher and Rackwitz [2], and Joanni and Rackwitz [3] compute the expected values of failure cost and

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benefits of a deteriorating system that is being inspected at regular time intervals followed by repairs. The probability of repair or no repair after inspection is computed based on either age or condition of the system. Similarly, Noortwijk [4], Noortwijk and Weidi [5] and Weidi et al. [6] proposed equations for expectation and variance of life-cycle costs for infinite time horizon. Noortwijk and Frangopol [7] compare the results from a renewal model and simulation based model. Several researchers have also developed methods that are not based on renewal theory. Typically, such methods are either purely based on monte-carlo simulation [8–16] or are analytical methods based on some simplifying assumptions. Simulations based methods are widely applicable but can be computationally expensive. Analytical methods not based on renewal theory [17–24] are typically unable to model deterioration as a stochastic process and mostly a deterministic function of time is used to represent deterioration. Furthermore, it is difficult to consider both service ability and ultimate types of failures using such methods.

This paper proposes a novel renewal theory based LCA model (RTLCA) for deteriorating systems. The proposed model provides equations to compute the instantaneous probability of being in service, the expected values and variances of availability, age, benefit, and costs of operation and failures of the system for a finite time horizon. The model accounts for both serviceability and ultimate failures. Although many of the mentioned concepts exist in the available renewal models, in the past they have been discussed primarily in the context of infinite time horizon which is not representative of an engineering system's life-span. In this paper, we derive all the equations for finite time horizon. In our knowledge, the proposed concepts and treatment of instantaneous probability of being in service and age is novel in LCA. The repair durations are considered dependent on the level of damage and are not considered negligible as has often been found in existing models. The presented formulation is made general so that equations are not dependent on any specific method to model deterioration as long as certain renewal probabilities and density functions can be computed. An example is presented to illustrate the proposed RTLCA formulation where the life-cycle of a reinforced concrete (RC) bridge is analyzed accounting for the deterioration caused by earthquakes and corrosion. The occurrence of earthquakes is modeled using a time-dependent stochastic process accounting for both main shocks and aftershocks. The example considers delays in repair after earthquakes and the damage accumulated during this delay due to aftershocks. Corrosion is modeled as a function of time with a random initiation time.

This paper is organized into six sections including this introduction. The second section describes the events typically observed in the life-cycle of an engineering system and introduces a few definitions used in the paper. The third section develops the equations for computing various LCA variables based on renewal theory. The fourth section briefly describes an existing stochastic deterioration model that can be used for RTLCA. The fifth section uses the stochastic deterioration model and the proposed RTLCA formulation to analyze the life-cycle of an example RC bridge. The sixth section presents the conclusions derived from this work.

2. Life-cycle of an engineering system

Fig. 1 shows the various events in the life-cycle of an engineering system that is experiencing deterioration. The state of the system at a given time t is described in terms of the probability of ultimate failure $P_f(t)$ of the system given that a load acts on the system at time t. Changes in $P_f(t)$ occur in the form of discrete or continuous increments. Discrete increments are due to shocks that cause sudden changes in the system properties. Continuous increments in $P_f(t)$ are due to a gradual deterioration of the system properties due to phenomena like corrosion of steel, alkali-silica reactions, delayed-ettringite formation, creep, etc.

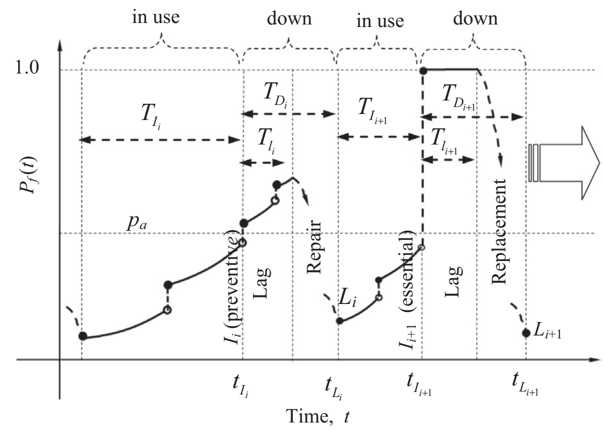

Fig. 1. Life-cycle of an engineering system.

Fig. 1 shows that an engineering system experiences alternating phases of being in use and in down-time. A system is said to be in use at time t if the system is functioning at that time. On the other hand, a system is said to be down or experiencing down-time if the system is either abandoned or removed from the service for repairs or replacement. In this paper, we call the start of a down-time as an *intervention* (I). The down-time of a system ends when the repair or replacement is complete and the system starts functioning again. In this paper, we call this event *renewal* (L). As mentioned earlier, interventions can be preventive or essential. Preventive interventions are typically made when a pre-determined safety related intervention criterion is met. Some examples of intervention criteria are: the exceedance of a threshold intensity of the applied load, a serviceability type failure such as exceedance of a threshold level for damage or $P_f(t)$, and reaching a pre-determined time elapsed since previous renewal (like in the case of a scheduled maintenance). Fig. 1 shows that the ith intervention I_i that occurs at time t_{I_i} is preventive and is conducted because $P_f(t) \geqslant p_a$. The figure also shows that I_{i+1} is an essential intervention and occurs because the system experiences an ultimate failure at time $t_{I_{i+1}}$ because of which $P_f(t)$ jumps to 1.0. The corresponding renewal events L_i and L_{i+1} occur at time t_{L_i} and $t_{L_{i+1}}$, respectively. In the figure, T_{I_i} is the time interval between L_{i-1} and I_i and T_{D_i} is the down-time following I_i.

For some systems, deterioration does not progress during the down-time because the system is removed from service and it is immediately repaired. However, in some cases (as shown in the figure) the actual repair work may not begin immediately at t_{I_i} and a lag period (T_{l_i} following I_i and $T_{l_{i+1}}$ following I_{i+1}) may exist during which the deterioration process may continue. Generally, this is the time required for the mobilization of the required resources. For example, an infrastructure that has been closed due to damage from an earthquake is still exposed to aftershocks before the repairs or replacement might take place. In such cases, the lag period may significantly affect the LCA and hence must be considered.

3. Economic feasibility considerations for a system

The costs incurred in the life-cycle of the system after its initial construction can be grouped into either cost of operation $C_{Op}(t)$ or failure losses $C_L(t)$. The cost $C_{Op}(t)$ is the total cost of repairs and replacement of the system following the serviceability and ultimate failures in order to operate the system up to time t. The