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Probabilistic analysis at the serviceability limit state of two neighboring strip footings resting on a spatially random soil

Ashraf Ahmed [a], Abdul-Hamid Soubra [b,*]

[a] University of Aswan, Abul-Rish kebly, Aswan, Egypt
[b] University of Nantes, Bd. de l'université, BP 152, 44603 Saint-Nazaire Cedex, France

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ABSTRACT

The computation of the failure probability of geotechnical structures considering the soil spatial variability is generally performed using Monte Carlo Simulation (MCS) methodology. This method is very time-consuming especially when computing a small failure probability. As an alternative, Subset Simulation (SS) approach was proposed by Au and Beck [3] to efficiently calculate the small failure probability. In the present paper, a more efficient approach called "improved Subset Simulation (iSS)" approach is employed. In this method, the first step of the SS approach is replaced by a conditional simulation in which the samples are generated outside a hypersphere of a given radius. The efficiency of the iSS approach is illustrated here through the probabilistic analysis at the serviceability limit state (SLS) of two neighboring strip footings resting on a soil with 2D spatially varying Young's modulus. The system response is the differential settlement between the two footings. The probabilistic results have shown that the probability P_e of exceeding a tolerable differential settlement computed by the iSS approach is very close to that calculated by the MCS methodology applied on the original deterministic model. The results have also shown that the use of the iSS approach has reduced the number of calls of the deterministic model by about 50% with respect to the SS approach.

1. Introduction

The classical Monte Carlo Simulation (MCS) methodology is generally used to calculate the failure probability of geotechnical problems involving random fields (e.g. [8,14,5] at ULS and [7,9] at SLS analysis). In these studies, only the mean value and the standard deviation of the system response were extensively investigated. This is because MCS requires a large number of calls of the deterministic model for the computation of the small failure probabilities. As alternative to MCS methodology, the Subset Simulation (SS) approach was proposed by Au and Beck [3] to calculate the small failure probability. The first step of the SS method is to generate a given number of realizations of the uncertain parameters using the classical MCS technique. The second step is to use the Metropolis–Hastings (M–H) algorithm to generate realizations in the direction of the limit state surface. This step is repeated until reaching the limit state surface. It should be emphasized here that in case of a small failure probability, SS requires the repetition of the second step several times to reach the limit state surface. This leads to a high number of calls of the deterministic model and consequently a high computational time. To reduce the computation time of the SS approach, Defaux et al. [6] proposed a more efficient approach called "improved Subset Simulation (iSS)". In this method, the first step of the SS approach was replaced by a conditional simulation. In other words, instead of generating realizations directly around the origin by the classical MCS, the realizations are generated outside a hypersphere of a given radius R_h. This reduces the number of realizations which are not located in the failure zone. Consequently, the number of realizations required to reach the limit state surface is significantly reduced. Notice that Defaux et al. [6] have employed the iSS to calculate the failure probability in the case where the uncertain parameters are modeled by random variables. In the present paper, the iSS is employed in the case where the uncertain parameters are modeled by random fields. This method is illustrated herein through the computation of the probability (P_e) of exceeding a tolerable differential settlement between two neighboring strip footings resting on a soil with a 2D spatially varying Young's modulus. The Young's modulus is modeled by a random field. The footings are subjected to central vertical loads with equal magnitude. The random field is discretized into a finite number of random variables using the Karhunen–Loeve (K–L) expansion. The differential settlement between the two footings was used to represent the system response. The deterministic model used to compute the system response is based on numerical simulations using the commercial software FLAC.

This paper is organized as follows: a brief review of the SS approach and the Karhunen–Loeve expansion method is first

* Corresponding author. Tel.: +33 240905108; fax: +33 240905109.
E-mail address: Abed.Soubra@univ-nantes.fr (A.-H. Soubra).

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presented. Then, the iSS approach and its implementation in the case of random field problems are presented. This is followed by the illustration of the efficiency of the iSS approach through the probabilistic analysis of two neighboring strip footings resting on a spatially varying soil. The paper ends with a conclusion of the main findings.

2. Review of subset simulation approach

Subset simulation was proposed by Au and Beck [3] to compute the small failure probabilities. The basic idea of the subset simulation approach is that the small failure probability can be expressed as a product of larger conditional failure probabilities. Consider a failure region F defined by the condition $G < 0$ where G is the performance function and let $(s_1, \ldots, s_k, ..., s_{Nt})$ be a sample of N_t realizations of a vector 's' composed of M random variables. It is possible to define a sequence of nested failure regions $F_1, \ldots, F_j, ..., F_m$ of decreasing size where $F_1 \supset \ldots \supset F_j \supset \ldots \supset F_m = F$ (Fig. 1). An intermediate failure region F_j can be defined by $G < C_j$ where C_j is an intermediate failure threshold whose value is larger than zero. Thus, there is a decreasing sequence of positive failure thresholds $C_1, \ldots, C_j, ..., C_m$ corresponding respectively to $F_1, \ldots, F_j, \ldots, F_m$ where $C_1 > \ldots > C_j > \ldots > C_m = 0$. In the SS approach, the space of uncertain parameters is divided into a number m of levels with equal number N_s of realizations $(s_1, \ldots, s_k, ..., s_{Ns})$ where $N_t = N_s \times m$. An intermediate level j contains a safe region and a failure region defined with respect to a given failure threshold C_j. The conditional failure probability corresponding to this intermediate level j is calculated as follows:

$$P(F_j|F_{j-1}) = \frac{1}{N_s}\sum_{k=1}^{Ns} I_{F_j}(s_k) \tag{1}$$

where $I_{F_j}(s_k) = 1$ if $s_k \in F_j$ and $I_{F_j}(s_k) = 0$ otherwise. Notice that in the SS approach, the first N_s realizations are generated using MCS methodology according to a target PDF P_t. The next N_s realizations of each subsequent level are obtained using Markov chain method based on Metropolis–Hastings (M–H) algorithm according to a proposal PDF P_p. Notice that a modified M–H algorithm was proposed by [15]. This modified algorithm was used in this paper to generate the realizations of level j ($j = 1, 2, \ldots, m$).

The failure probability $P(F) = P(F_m)$ of the failure region F can be calculated from the sequence of conditional failure probabilities as follows:

$$P(F) = P(F_1)\prod_{j=2}^{m} P(F_j|F_{j-1}) \tag{2}$$

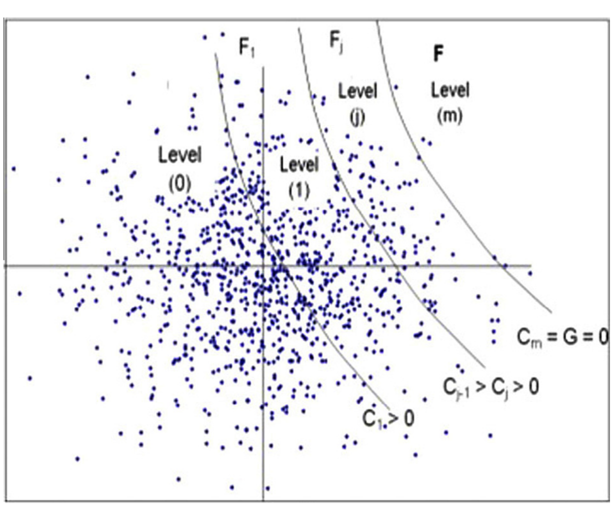

Fig. 1. Nested failure domain.

where $P(F_1)$ is the failure probability corresponding to the first level of the SS approach, m is the number of levels required to reach the limit state surface and $P(F_j|F_{j-1})$ is an intermediate conditional failure probability. This equation can be regarded as a system consisting of m components (related to the m failure regions $F_1, \ldots, F_j, \ldots, F_m$) connected in parallel. Consequently, the failure probability of the failure region F is the intersection of all conditional failure probabilities of the failure regions $F_1, \ldots, F_j, \ldots, F_m$. Thus, the failure probability $P(F)$ is:

$$P(F) = P(\cap_{j=1}^{m} F_j) \tag{3}$$

where

$$P(\cap_{j=1}^{m} F_j) = P(F_m \big| \cap_{j=1}^{m-1} F_j) x P(\cap_{j=1}^{m-1} F_j) = P(F_m|F_{m-1}) x P(\cap_{j=1}^{m-1} F_j)$$
$$= \ldots = P(F_1)\prod_{j=2}^{m} P(F_j|F_{j-1}) \tag{4}$$

It should be noticed here that the computation of the failure probability $P(F)$ may be determined using alternatively one of the two following procedures. The first procedure consists in prescribing a sequence of $C_1, \ldots, C_j, \ldots, C_m$ so that $C_1 > \ldots > C_j > \ldots > C_m = 0$ and then, calculating the different values of $P(F_j/F_{j-1})$ at the different levels using Eq. (1). The second procedure consists in prescribing a constant conditional failure probability $P(F_j/F_{j-1})$ for the different levels and then, calculating the different C_j values corresponding to these levels. The value of C_j of level j is the one for which the ratio between the number of realizations for which $G < C_j$ and the number of realizations N_s of this level (which is identical for the different levels), is equal to the prescribed value $P(F_j/F_{j-1})$. In this paper, the second procedure is used. Notice that, for simplicity in notations, the constant conditional failure probability $P(F_j/F_{j-1})$ will be referred to as p_0 later on. The algorithm of the SS approach can be described by the following steps:

(1) Generate a realization of the vector 's' of M random variables by MCS according to the target PDF P_t.
(2) Using the deterministic model, calculate the system response corresponding to this realization.
(3) Repeat steps 1 and 2 until obtaining a prescribed number N_s of realizations of the vector 's' and the corresponding system response values. Then, evaluate the corresponding values of the performance function to obtain the vector $G_0 = \{G_0^1, \ldots, G_0^k, \ldots, G_0^{Ns}\}$. Notice that the values of the performance function of the different realizations are arranged in an increasing order in the vector G_0. Notice also that the subscripts '0' refer to the first level (level 0) of the subset simulation.
(4) Prescribe a constant conditional failure probability p_0 for all the failure regions F_j ($j = 1, \ldots, m-1$) and evaluate the first failure threshold C_1 which corresponds to the failure region F_1 where C_1 is equal to the $[(N_s \times p_0) + 1]^{\text{th}}$ value in the increasing list of elements of the vector G_0. This ensures that the value of $P(F_1)$ will be equal to the prescribed p_0 value.
(5) Among the N_s realizations, there are $[N_s \times p_0]$ ones whose values of the performance function are less than C_1 (i.e. they are located in the failure region F_1). These realizations are used as 'mother realizations' to generate additional $[(1-p_0)$-$N_s]$ realizations of the vector 's' using Markov chain method based on Metropolis–Hastings algorithm. These new realizations are located in the second level (level 1 in Fig. 1).
(6) The values of the performance function corresponding to the realizations obtained from the preceding step are listed in an increasing order and are gathered in the vector of performance function values $G_1 = \{G_1^1, \ldots, G_1^k, \ldots, G_1^{Ns}\}$.
(7) Evaluate the second failure threshold C_2 as the $[(N_s \times p_0) + 1]^{\text{th}}$ value in the increasing list of elements of the vector G_1.