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Value of information analysis with structural reliability methods

Daniel Straub *

Engineering Risk Analysis Group, Technische Universität München, Germany

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ABSTRACT

When designing monitoring systems and planning inspections, engineers must assess the benefits of the additional information that can be obtained and weigh them against the cost of these measures. The value of information (VoI) concept of the Bayesian statistical decision analysis provides a formal framework to quantify these benefits. This paper presents the determination of the VoI when information is collected to increase the reliability of engineering systems. It is demonstrated how structural reliability methods can be used to effectively model the VoI and an efficient algorithm for its computation is proposed. The theory and the algorithm are demonstrated by an illustrative application to monitoring of a structural system subjected to fatigue deterioration.

1. Introduction

When it is required to make decisions under uncertainty and risk, one often has the possibility to gather further information prior to making the decision. Such information reduces the uncertainty and thus facilitates improved decision making. This explains the success of structural health monitoring (SHM), advanced inspection methods, remote sensing and other monitoring techniques for civil infrastructures, to which I will refer collectively as monitoring systems.

As experienced engineers are well aware, collecting the information comes at a price that is not always justified by its benefit. Unfortunately, this is often discovered only after the installation of a monitoring system. A mathematical framework exists for quantitatively assessing the benefit of a monitoring system prior to installing it: the value of information (VoI) analysis from Bayesian statistical decision theory [1–3] that has been considered by civil and structural engineers since the early 1970s [4]. The late Prof. Wilson Tang was one of the first to notice the potential of Bayesian methods and VoI concepts to optimize engineering decisions [5–7]. In his paper published in 1973 [5], he described Bayesian updating of probabilistic models of flaws with inspection results, which preceded the optimization of inspections in aircraft and offshore structures subject to fatigue deterioration in the 1970s and 80s [8–12]. These works were among the first applications of Bayesian decision analysis for optimizing the collection of information in an industrial context. Similar efforts were made in the field of transportation infrastructure management, based on Markovian deterioration models [13]. In recent years, the optimization of monitoring systems through explicit computation of the VoI has found increased interest in various fields of civil and infrastructure engineering. Explicit computation of the VoI for optimizing inspections and structural health monitoring in deteriorating structures was proposed in [14–18]. Optimization of sensor placement based on VoI has been studied in [19]. In geotechnical engineering, which has always been strongly relying on monitoring, the effect of information quality has been investigated [20]; an explicit quantification of the VoI for head monitoring of levees is described in [21]. In the field of natural hazards, the VoI concept has been applied for prioritizing post-earthquake inspections of bridges [22] and for quantifying the value of improved climate models when designing offshore structures against extreme wave loads [23]. VoI analysis is and has been applied in many other fields of engineering and science, including oil exploration [24] and environmental health risk management [25].

Determining the VoI requires significant modeling and computational efforts. Computationally efficient evaluations of the VoI was considered mainly in the field of machine learning and artificial intelligence [19,26–28]. In these areas, prediction models used for the VoI computations are typically based on known probabilistic dependences among a potentially large number of random variables. In contrast, in infrastructure and civil engineering, prediction models are often based on advanced physically-based models, which describe the monitored phenomena. As an example, when planning the monitoring of a bridge, one can make use of detailed mechanical models of the structure. Furthermore, the monitoring is often installed not to guide the every-day operation of the system, but for early detection of deterioration or damages that may

* Tel.: + 49 8928923051.
E-mail address: straub@tum.de
URL: http://www.era.bgu.tum.de

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impair the safety of the system. These applications motivate the combination of the VoI concept with structural reliability methods, which were developed to efficiently compute the probability of system failure via advanced physically-based models.

This paper presents the modeling and computation of VoI based on structural reliability methods. A modeling framework is proposed, which is especially suitable when probabilistic physically-based models of the monitored systems and processes are available, e.g. in structural engineering applications. On this basis, a computationally efficient algorithm is developed for estimating the VoI. The framework and the algorithm are illustrated through an application to monitoring of a structure subject to fatigue deterioration, which demonstrates the effectiveness and efficiency of the proposed approach. The paper closes with a discussion on the difficulties encountered in determining the VoI in realistic engineering problems.

2. Value of information analysis

2.1. Decision-theoretic framework

As a premise, I assume that all consequences (costs of monitoring, mitigation actions as well as failure consequences) can be expressed either in monetary values or in a common measure of utility U. I adopt the classical expected utility framework [29] according to which an optimal decision under uncertainty is the one maximizing the expected utility E[U]. For simplicity, I further restrict the presentation to situations in which all consequences can be expressed as monetary costs C and in which utility is proportional to $-C$, corresponding to a risk-neutral decision maker. The optimal decision is thus the one that minimizes the expected cost E[C]. It is straightforward to adapt the methods presented in this paper to the case of a risk-averse decision maker or to situations with non-monetary consequences, if preferences of the decision maker can be expressed through utility functions.

Following the classical structural reliability modeling framework [30], the uncertainty associated with the phenomena under consideration is characterized by a vector $\mathbf{X}$ of random variables. The relation between $\mathbf{X}$ and the events of interest is a deterministic one, e.g. the failure event is described through the limit state function $g_F(\mathbf{X})$ as $F = \{g_F(\mathbf{X}) \leqslant 0\}$. In this framework, model uncertainties are included through additional random variables in $\mathbf{X}$.

In a classical decision analysis under uncertainty, the goal is to identify the actions a that minimize E[C], e.g. the maintenance and repair actions a that ensure an optimal balance between the cost of a and the risk associated with failure. Additionally, information can be collected prior to making the action decision a. Therefore, a so-called test decision e is made on what information to collect (e stands for experiments). This is, e.g., a decision on the design of a monitoring system or a decision on the inspection schedule. The extended decision problem is to find the combination of monitoring decision e and action decision a that minimizes E[C]. This problem is known in the literature as *preposterior decision analysis* [4]. These problems can be graphically modeled through decision trees and influence diagrams, Fig. 1. The decision tree explicitly depicts all possible states of random variables and decisions. In contrast, the influence diagram provides a more concise representation, which additionally reflects the causal relations between the random variables and the decisions. Implementations of the influence diagram for computing the VoI can be found in [16,31].

This paper focuses on the computation of the value of information (VoI) of a given monitoring system. The optimization of the monitoring system (the test decision e) is not explicitly considered. However, the VoI is the total expected net benefit of a given monitoring system and is thus the central part of any preposterior decision analysis. The optimal monitoring system is the one maximizing the VoI minus the cost of monitoring.

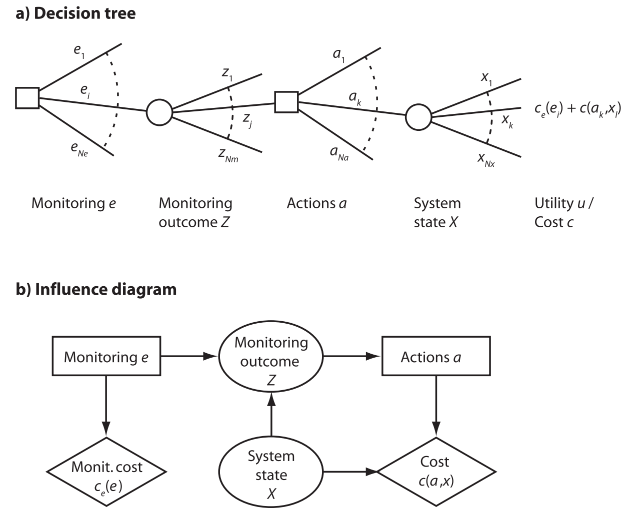

Fig. 1. The basic decision problem when planning monitoring and inspection measures: (a) decision tree and (b) corresponding influence diagram. Here it is assumed that the cost of monitoring $c_e(e)$ and the cost of the action and system state $c(a,x)$ are additive.

In the following, the optimization of the decision a is presented prior to considering the monitoring results. This follows the logic that monitoring results enable improved action decisions and that their benefit can thus only be quantified when explicitly modeling the action decision.

2.2. Prior decision optimization

Before applying monitoring, the optimization of the decision a must be based on the prior knowledge, characterized through the prior probability distribution of $\mathbf{X}$. The prior optimization problem is:

$$a_{opt} = \arg\min_a \mathrm{E}_{\mathbf{X}}[c(a,\mathbf{X})] = \arg\min_a \int_{\mathbf{X}} c(a,\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) \mathrm{d}\mathbf{x}. \tag{1}$$

$c(a,\mathbf{x})$ is the cost associated with a given set of actions a and realization $\mathbf{x}$, and $\mathrm{E}_{\mathbf{X}}$ denotes the expectation with respect to $\mathbf{X}$. Throughout the paper I use the notation $\int_{\mathbf{X}} \mathrm{d}\mathbf{x} = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \mathrm{d}x_1 \ldots \mathrm{d}x_n$.

In engineering decision problems involving reliability, the consequences typically depend on discrete events describing the system state, such as failure F or a set of damage levels (e.g., in performance-based earthquake engineering). In the structural reliability framework, these events correspond to domains in the outcome space of $\mathbf{X}$. Let $E_1, E_2, \ldots, E_m$ denote the mutually exclusive, collectively exhaustive system states in the general case. (If only failure F is of interest, it is $E_1 = F$ and $E_2 = \overline{F}$.) The optimization problem can then be written as

$$a_{opt} = \arg\min_a \sum_{i=1}^{m} c_{E_i}(a) \Pr(E_i). \tag{2}$$

Here, $c_{E_i}(a)$ is the cost associated with event E_i and decision a. Let C_{prior} denote the expected cost associated with this optimal decision a_{opt}, i.e.

$$C_{prior} = \min_a \sum_{i=1}^{m} c_{E_i}(a) \Pr(E_i) = \sum_{i=1}^{m} c_{E_i}(a_{opt}) \Pr(E_i). \tag{3}$$