Structural Safety 47 (2014) 39–47

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Identification of critical samples of stochastic processes towards feasible structural reliability applications

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ARTICLE INFO

Article history:
Received 11 February 2012
Received in revised form 21 October 2013
Accepted 21 October 2013
Available online 26 November 2013

Keywords:
Stochastic process
Critical excitation
Reliability analysis
Importance sampling
Image processing
Pattern recognition
Identification problem

ABSTRACT

This paper contributes to the structural reliability problem by presenting a novel approach that enables for identification of stochastic oscillatory processes as a critical input for given mechanical models. The proposed method is based on a graphical representation of such processes utilizing state of the art image processing and pattern recognition techniques, leading to a set of finite rules that consistently identifies those realizations of stochastic processes that would lead to a critical response of a given mechanical model. To examine the validity of the suggested method, large sets of realizations of artificial non-stationary processes were generated from known models, several criteria for critical response were formulated and the results were statistically evaluated. The promising results suggest important applications that would dramatically decrease computational costs e.g. in the field of probabilistic seismic design. Further examination may lead to a formulation of a new class of importance sampling techniques.

1. Introduction

The necessity for adopting fully probabilistic design concepts has become imperative when considering static loads [1,2]. On the other hand, structural dynamics is still far from a practical utilization of such concepts despite cheap contemporary computational costs. This is mainly due to the uncertain nature of environmental loadings that have to be modeled as time-varying phenomena, represented in this paper by non-stationary stochastic oscillatory process as defined by Priestley [3].

It is a well-accepted fact that structures respond in a very uncertain manner to probabilistically different ground motion events while there is very limited a priori knowledge on the structural behavior. An implication is the necessity to perform the structural analysis for each realization of the event separately, which makes the Monte-Carlo based reliability analysis computationally unfeasible for some realistic assumptions, i.e. small probabilities and large sample sizes.

There have been several recent attempts to avoid such reliability problems in their full form. Moustafa [4] proposed a framework for deriving optimal earthquake loads expressed as a Fourier series. More recently, critical excitation methodologists propose to identify critical frequency content of ground motions maximizing the mean earthquake energy input rate to structures (for details see e.g. [5]). From a different perspective, Barbato et al. [6] approximate the first passage problem by formulating exact closed form solutions for the spectral characteristics of random processes. Macke et al. [7] present an importance sampling technique for randomly excited dynamical systems.

Authors of this paper attempt to maintain the up-to-date conceptually correct fully probabilistic concept [8] while reducing the number of required analyses by means of the proposed identification framework. It is based on a non-traditional assumption that there exists a finite set of rules capable of classifying synthetic samples of stochastic processes according to their importance as a critical input for a given mechanical model. Whether such sets of rules could be formulated for an arbitrary system remains an open problem for further research.

2. Development

The identification strategy follows a transparent image processing paradigm completely independent of structural dynamics, thus representing a nontraditional option in the field. The reason behind such argument is experimental, aiming at delivering a simple and wide-purpose method.

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http://dx.doi.org/10.1016/j.strusafe.2013.10.005
0167-4730/

The main objective can be formulated as follows: find the critical realization of a stochastic process from a target sample set $\boldsymbol{S}$ under defined critical response criteria. In the following text the symbol $\rightarrow$ designates higher order mapping function, e.g. $x_0 \rightarrow f(x_0)$.

Suggested Small Training Set (STS) input format:

(i1) Finite set $\boldsymbol{S}$ of 1-dimensional stochastic process realizations $\boldsymbol{r_i}$

$$\boldsymbol{r_i} \in \langle v_1, v_2, \ldots, v_t \rangle \tag{1}$$

$$\boldsymbol{S} \in \langle \boldsymbol{r_1}, \boldsymbol{r_2}, \ldots, \boldsymbol{r_n} \rangle \tag{2}$$

where v_i can be an arbitrary value, e.g. acceleration values recorded in t time steps. The notation n is used for the size of $\boldsymbol{S}$.

(i2) Arbitrary deterministic solver, typically extremely expensive computational numerical integration, e.g. FEM, or reduced meta-models such as MOR and POD [9],

$$F(\boldsymbol{r_i}) \stackrel{N\,Solve}{\rightarrow} y_i \tag{3}$$

returning an arbitrary scalar response quantity y_i, e.g. a peak displacement.

(i3) Arbitrary algorithm for 2-dimensional graphical representation $\boldsymbol{G}$ of r_i. Among the two general options maintaining the physicality of the $\boldsymbol{G}$ product is the evolutionary spectra [10] or the wavelet–vector coefficients based scalogram [11] *wd*. The latter is used in this paper due to preferable computational complexity, basic principles demonstrates sample patterns at Figs. 1 and 2.

$$wd(\boldsymbol{r_i}) \stackrel{maps}{\rightarrow} \boldsymbol{G_i} = \begin{bmatrix} c_{1,1}^{(i)} & \cdots & c_{t,1}^{(i)} \\ \vdots & \ddots & \vdots \\ c_{1,o}^{(i)} & \cdots & c_{t,o}^{(i)} \end{bmatrix} \tag{4}$$

Here the wavelet vector coefficients (1,…,o) are plotted as rows of colorized rectangles $c_{t,o}$, in which large absolute values are shown darker and each subsequent row corresponds to different wavelet index specifications. Note that the actual choice of the mapping algorithm in this step is quite immaterial for proper functioning of the method as long as it allows a time-frequency decomposition of the signal.

(i4) Parameters vector and admissible intervals $f\langle 1\ldots(t \times o)\rangle$, $m\langle 1\ldots n\rangle$, $p\langle 1\ldots m/2\rangle$, $q\langle 1\ldots m\rangle$

Proposed STS strategy steps:

(s1) Construct a training subset $\boldsymbol{s}$ randomly sampled from $\boldsymbol{S}$ having the length $m \ll n$.

$$\boldsymbol{s} \in \langle r_1, r_2, \ldots, r_m \rangle \tag{5}$$

(s2) Solve the training subset $\boldsymbol{s}$:

$$\langle F(r_1), F(r_2), \cdots, F(r_m) \rangle \stackrel{yields}{\rightarrow} \boldsymbol{s_F} \in \langle y_1, y_2, \cdots, y_m \rangle \tag{6}$$

(s3) Create ranked minimum and maximum sets $\boldsymbol{S_{F,min}}$ and $\boldsymbol{S_{F,max}}$:

$$\boldsymbol{s}_{F,\min} = \langle 1, \ldots, p \rangle^{th} \text{ smallest elements in } \boldsymbol{s_F} \tag{7}$$

$$\boldsymbol{s}_{F,\max} = \langle 1, \ldots, p \rangle^{th} \text{ largest elements in } \boldsymbol{s_F} \tag{8}$$

(s4) Transform $\boldsymbol{r_i}$'s corresponding to $\boldsymbol{S_{F,min}}$ and $\boldsymbol{S_{F,max}}$ into graphical representation

$$wd(\boldsymbol{r}_{s_{F,\min}}), wd(\boldsymbol{r}_{s_{F,\max}}) \stackrel{yields}{\rightarrow} \langle \boldsymbol{G}_{\min,1}, \ldots, \boldsymbol{G}_{\min,p} \rangle, \langle \boldsymbol{G}_{\max,1}, \ldots, \boldsymbol{G}_{\max,p} \rangle \tag{9}$$

(s5) Find a finite set of rules $\boldsymbol{R}$ such that consistently maps the s4) products to the corresponding few important (i.e. maximal or critical) response criteria. Note that a one-to-one correspondence is likely unfeasible and the search domain can be effectively narrowed by ignoring the pixels with constant or random behavior. A simple specific form of $\boldsymbol{R}$ can be attained by calculating the 2-dimensional correlation pattern $\boldsymbol{P}$ using the $\boldsymbol{G_{min}}$ and $\boldsymbol{G_{max}}$:

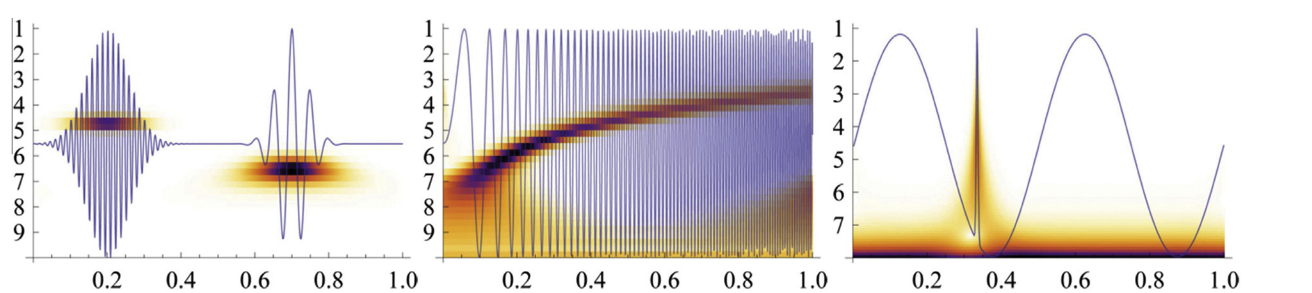

Fig. 1. Relationship between the signal and the $\boldsymbol{G}$ pattern. Horizontal axes represents time and is joined for both the signal and $\boldsymbol{G}$. Vertical axis of $\boldsymbol{G}$ represents the equivalent scales (octaves or frequency bands).

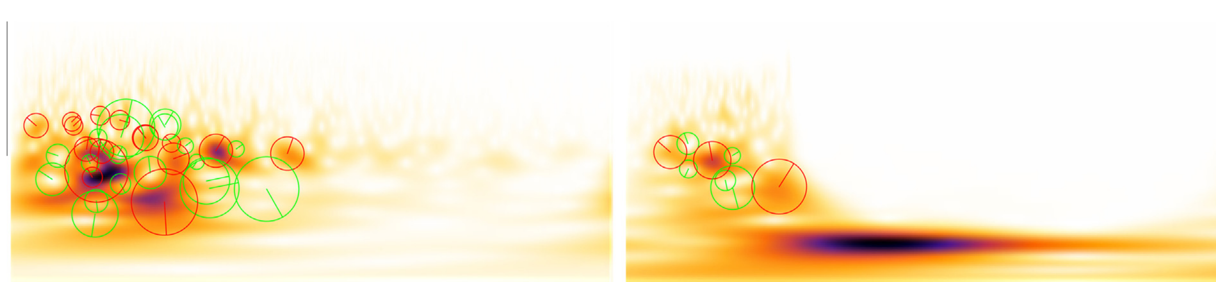

Fig. 2. Graphical representation ($\boldsymbol{G}$) of L1 (left) and L2 (right) in the form of Wavelet Scalogram and visualized detected keypoints ($\boldsymbol{R}$) described by radius, orientation and contrast sign (circle, rotation and color). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)