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Quantile value method versus design value method for calibration of reliability-based geotechnical codes

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ABSTRACT

This paper compares two methods for geotechnical reliability code calibration, namely the well known design value method (DVM) based on first-order reliability method and a recently developed method based on quantile, called the quantile value method (QVM). The feasibility of calibrating a single partial factor to cover the wide range of coefficients of variation (COVs) commonly encountered in geotechnical designs is studied. For analytical tractability, a simple design example consisting of one resistance random variable and one load random variable is first examined. A resistance factor is first calibrated using a single calibration case associated with a typical COV. The objective is to evaluate the departure from the target reliability index analytically when this calibrated resistance factor is applied to validation cases associated with a range of COVs. The results show that QVM is more robust than DVM in terms of achieving a more uniform reliability level over a range of COVs. Two realistic geotechnical design examples are studied to demonstrate that the theoretical insights garnered in the simple analytical example are applicable.

1. Introduction

The coefficients of variation (COVs) for geotechnical parameters are not constant. Taking the undrained shear strength (s_u) of a clay as an example, measurement errors for undrained shear strengths obtained from unconfined compression (UC) tests are typically larger, compared to s_u obtained from more sophisticated tests such as isotropically consolidated undrained compression (CIUC) tests. Spatial averaging over a large volume of soil mass may also significantly reduce the COV in s_u [1]. The averaging volume is problem dependent. In addition, there are various methods of estimating s_u. For example, s_u can be estimated from the preconsolidation stress or from the liquidity index. Different transformation equations are needed to convert the measured parameter (preconsolidation stress or liquidity index) to the desired design parameter (s_u). The transformation uncertainties can vary significantly as well [2]. For example, the s_u versus preconsolidation stress transformation usually is associated with less transformation uncertainty than the s_u versus liquidity index transformation. Geotechnical models, such as the classical limit equilibrium models for pile capacity, are not exact. Construction effects cannot be readily modeled or quantified to say the least. Construction effects can be significant in geotechnical engineering. Model factors are needed to relate somewhat idealized calculations with actual measured capacities. It is well established that model factors are random variables, typically lognormally distributed. The mean and COV of a model factor are typically obtained from calibration with field measurements (e.g., pile load test database). These statistics may change depending on the database, even for the same problem and the same calculation model.

The issue of COVs varying over a wide range is also often encountered in geotechnical design practice, because soil is a natural material and there is a diversity of testing methodologies developed to suit different site conditions. In contrast, concrete and steel are manufactured and testing methodologies are accordingly more standardized. Hence, structural design practice does not need to contend with COVs varying over a wide range. This issue must be dealt with in geotechnical reliability-based design, although it poses a significant challenge. To elaborate on this challenge, consider a simple pile design problem involving two variables, the resistance Q and the load L. Let V_Q and V_L be the COVs of the resistance and load, respectively. Assume that $V_L = 0.15$ is constant, but V_Q is not constant. Let scenario A be a case where a detailed site investigation and extensive load tests have been conducted. As a result, V_Q is small and equal to 0.2. Scenario B is a case where the site investigation is cursory and no load test is conducted. As a result, V_Q is large and equal to 0.45. It is evident that a set of constant load and resistance factors (or partial factors) cannot maintain a uniform reliability level over these two disparate

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scenarios. The challenge is obviously non-existent if one adopts a full probabilistic approach, rather than a simplified reliability-based design approach. It is assumed in this paper that the former is not acceptable to practitioners at the present moment, which is indeed the case in the geotechnical engineering community.

In this paper, a method named "quantile value method (QVM)" for calibrating partial factors is presented. This method is based on the quantile-based theoretical approach developed in [3]. The name QVM is herein selected to differentiate from the more widely known "design value method (DVM)" [4,5] based on the first-order reliability method (FORM) [6]. Both DVM and QVM adopt conservative design locations situated on the limit state line, but DVM adopts the FORM design point, while QVM adopts a design location that has not been explored in literature thus far. In this study, DVM and QVM will be compared using a simple geotechnical pile design involving only two random variables. For this simple example, exact solutions for both DVM and QVM are available, so the comparison can be made analytically and geometric interpretations can be presented visually in the standard Gaussian space. The comparison focuses on the ability to maintain a uniform reliability level over a wide range of validation design scenarios, such as different COVs, using a single prescribed number (resistance factor or quantile). The analytical comparison will be mostly limited to the case where the prescribed number is calibrated from a single design scenario, but validation would cover a number of design scenarios. Calibration involving multiple design scenarios will be addressed numerically in association with two realistic geotechnical design examples. It will be shown that most of the issues encountered for the realistic examples can be explained by the theoretical insights garnered in the simple analytical example.

2. Analytical example

The following simple example is adopted to compare DVM and QVM analytically. Consider a pile with axial resistance Q and subjected to axial load L. Q and L are independent and lognormally distributed with mean values (μ_Q, μ_L) and COVs (V_Q, V_L). The limit state function is defined to be $G = \ln(Q) - \ln(L)$. In the standard Gaussian space,

$$g(z_Q, z_L) = \lambda_Q + \xi_Q z_Q - \lambda_L - \xi_L z_L$$
$$\xi = \sqrt{\ln(1+V^2)}, \quad \lambda = \ln(\mu) - 0.5 \times \xi^2 \tag{1}$$

where λ and ξ are respectively the mean and standard deviation of the logarithm of the subscripted variable, and (z_Q, z_L) are jointly standard Gaussian. The safety ratio can be defined as

$$\mathrm{SR}(z_Q, z_L) = \frac{Q}{L} = \exp(\lambda_Q + \xi_Q z_Q - \lambda_L - \xi_L z_L) \tag{2}$$

Whenever $\mathrm{SR}(z_Q, z_L) < 1$, failure occurs, and vice versa.

Two cases would be considered: a calibration case and a validation case. The mean values and COVs for the calibration case are (μ_Q, μ_L) and COVs (V_Q, V_L), and those for the validation case are (μ'_Q and COVs (V'_Q Basically, the calibration case will be used to calibrate the partial factors (or load and resistance factors) to achieve a prescribed target reliability index of β_T. The validation case will be used to examine whether these partial factors indeed produce a design with an actual reliability index β'_A that is reasonably close to β_T.

The geometric interpretation is illustrated in Fig. 1. For this simple example, the limit state lines are linear in the standard Gaussian space. However, the limit state lines for the calibration case ($g = 0$) and validation case ($g' = 0$) are different and are in general not parallel to each other. The reason is that $V_Q \neq V'_Q$ and $V_L \neq V'_L$.

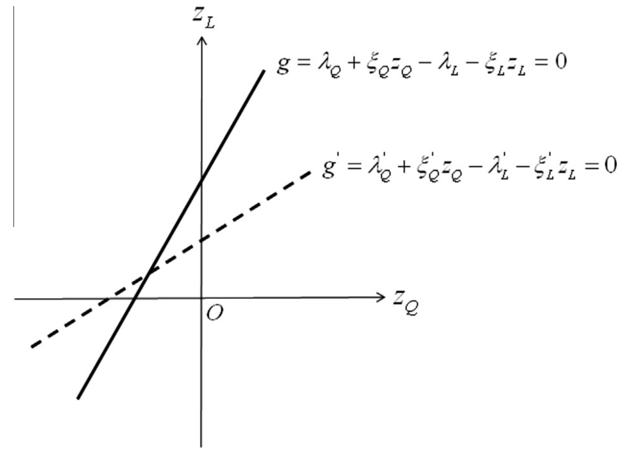

Fig. 1. Limit state lines of the calibration case ($g = 0$) and validation case ($g' = 0$).

3. Design value method and quantile value method

Reliability-based design is typically implemented in design codes using a set of partial factors (or load and resistance factors) that achieves the target reliability index β_T for the calibration case. However, this set of numbers is not unique. In the standard Gaussian space, any point on the "adjusted" limit state line $g(z) = \lambda_Q + \xi_Q z_Q - \lambda_L - \xi_L z_L = 0$ can be used to derive a set of partial factors. The adjusted limit state line is a limit state line with distance to the origin adjusted to β_T. The chosen point on the adjusted limit state line for evaluation of partial factors will be called the "design location" in this paper. The design location is not necessarily the same as the widely known FORM "design point": the design location can be anywhere on the limit state line $g = 0$, and the FORM design point is only a special case. It is the point on $g = 0$ nearest to the origin.

Before choosing a design location on the limit state line $g = 0$ for the calibration case, the limit state line must be adjusted to a distance of β_T from the origin to fulfill the target reliability index. This can be done by adjusting one or several of the design parameters $\{\mu_Q, \mu_L, V_Q, V_L\}$ or, equivalently, among $\{\lambda_Q, \lambda_L, \xi_Q, \xi_L\}$. It is usually impractical to adjust the COVs. The mean resistance can be increased by lengthening the pile and the mean load can be reduced by distributing the column load to multiple piles in a pile group. In the ensuing analysis, λ_L is adjusted to a value equal to $\lambda_Q - \beta_T(\xi_Q^2 + \xi_L^2)^{0.5}$. The adjusted design parameter will be called the "pivoting design parameter" in this paper. After the adjustment, the limit state line becomes

$$g(z) = \xi_Q z_Q - \xi_L z_L + \beta_T \sqrt{\xi_Q^2 + \xi_L^2} = 0 \tag{3}$$

Note that this adjustment is carried out on the calibration case, not on the validation case. There are many possible choices for the set of partial factors for the calibration case. The DVM and QVM are two special cases. Both methods select design locations on the adjusted limit state line but impose some restrictions explained below so that the resulting set is unique.

3.1. Design value method

The design value method (DVM) chooses the design location to be the FORM design point, which is the point on the adjusted limit state line that is closest to the origin (shown as z^* in the left plot of Fig. 2). It is also the most probable point on the adjusted limit state line for the calibration case. Direct calculation shows that the FORM design point has the following coordinates:

$$z_Q^* = \frac{-\beta_T \xi_Q}{\sqrt{\xi_Q^2 + \xi_L^2}}, \quad z_L^* = \frac{\beta_T \xi_L}{\sqrt{\xi_Q^2 + \xi_L^2}} \tag{4}$$