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# System reliability analysis of a stiffened panel under combined uniaxial compression and lateral pressure loads

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ABSTRACT

A system reliability analysis of an oil tanker bottom component which consists of a stiffened panel under combined uniaxial compression and lateral sea pressure loads is presented in this paper. The stiffened panel is idealized as a structural system composed by several stiffeners with attached plating in parallel. The structural capacity of each stiffener with attached plating or system component is described by a nonlinear finite element model, considering as failure criterion the buckling collapse under the combined uniaxial compression and lateral sea pressure loads. These load components are defined considering a typical seagoing operational condition of the oil tanker in ballast load. The uncertainty in the relevant design basic variables is quantified using stochastic models proposed in the literature. To efficiently solve the structural system reliability problem a Monte Carlo based reliability estimation method recently proposed is combined with a response surface method. The combination of these two methods has been shown to be an efficient technique to solve structural system reliability problems that involve computationally demanding numerical models to describe the structural capacity of the system components. Annual probabilities of buckling collapse failure of the stiffened panel are estimated using this solution technique. The effect of corrosion on the stiffened panel reliability is quantified. The importance of considering the lateral sea pressure and correlation between the local and global wave-induced loads in the reliability problem are evaluated.



## 1. Introduction

In general structural system reliability problems of practical engineering interest may involve several failure modes and a high number of basic random variables to properly describe the system structural capacity and the applied loads. The system failure criteria are very often associated with nonlinear structural behavior, requiring computationally demanding numerical approaches such as the nonlinear finite element analysis to accurately assess the structural capacity. It is known that the solution of these reliability problems is in general exceedingly difficult to obtain by conventional reliability methods as the first-order reliability method (FORM) or the second-order reliability method (SORM). The main reason is the high number of limit state functions and basic random variables as well as the computational effort that may be involved. It is recognized that the Monte Carlo simulation method is the most versatile solution method presently available, particularly for system reliability problems. With this method the system failure criteria are relatively easy to evaluate, almost irrespective of the complexity of the system and the number of basic random variables. However, the system failure probabilities are typically of rather small magnitude and therefore the computational effort involved in the Monte Carlo simulation may be prohibitive.

Recently, a new Monte Carlo based reliability estimation method for system reliability problems was proposed in [1]. The method provides accurate estimates of the system failure probability with low to moderate computational effort. It exploits the regularity of the tail probabilities to set up an approximation procedure to predict the far tail failure probabilities by extrapolation. This is achieved by formulating the reliability problem to depend on a parameter that scales the system failure probability, allowing accurate estimates to be obtained through crude Monte Carlo simulation with low to moderate computational cost. Approximating the system failure probability as a function of that parameter by a parametric function fitted to the crude Monte Carlo estimates, it is then possible to predict the system target failure probability by extrapolation. The method was first applied in [1] to small structural systems and later in [2] to a complex system, involving a high number of limit state functions and basic random variables. In [3] the method was combined with a response surface model to solve a system reliability problem that involved a nonlinear finite

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element model to describe the structural capacity of the system components. The buckling collapse strength reliability assessment of an oil tanker deck component in the form of a stiffened panel under uniaxial compression was adopted as numerical example. The stiffened panel was idealized as a structural system composed by several stiffeners with attached plating in parallel, with the structural capacity described by a nonlinear finite element model. It was shown that the Monte Carlo based reliability estimation method proposed in [1] combined with a response surface model is an efficient technique to solve structural system reliability problems that involve computationally demanding numerical models to describe the structural capacity.

In this study this solution technique is used to assess the buckling collapse strength reliability of an oil tanker bottom component which consists of a stiffened panel under combined uniaxial compression and lateral sea pressure loads, using an idealization of the stiffened panel as a structural system similar to that used in [3]. The reliability assessment considers annual probabilities of buckling collapse failure of the stiffened panel in a typical seagoing operational condition of the oil tanker in ballast load. Stochastic models proposed in the literature are used to quantify the uncertainty in the relevant design basic variables. The effect of corrosion on the stiffened panel reliability is quantified using an approach based on uniform thickness reduction of the attached plating and stiffeners cross-section. The contribution of the lateral sea pressure to the stiffened panel reliability is quantified. The dynamic or wave-induced component of the lateral sea pressure is considered to be correlated with the vertical wave-induced bending moment. The effect of this correlation on the stiffened panel reliability is also quantified.

## 2. Efficient system reliability estimation method

### 2.1. Modified system reliability problem

Consider a structural system composed by several structural components and that each component may have several failure modes. Let the system safety margin associated with a given failure mode be represented by:

$$M_i = G_i(X_1, \ldots, X_n) \tag{1}$$

with $G_i$, $i = 1,\ldots,m$, the limit state function that defines the safety margin $M_i$ as a function of a vector $\boldsymbol{X} = [X_1,\ldots,X_n]^T$ of basic random variables. Typically, these variables quantify the uncertain quantities involved in the reliability problem, such as material properties, dimensions of structural components and applied loads. The limit state function $G_i$ can be a very complicated function of the random vector $\boldsymbol{X}$. In some cases a close-form equation defining $G_i$ as a function $\boldsymbol{X}$ is not known and its evaluation requires the solution of computationally demanding numerical models (e.g., nonlinear finite element structural models). Failure in the mode $i$ of the system is assumed to occur when the safety margin associated with that mode satisfies $M_i = G_i(\boldsymbol{X}) \leqslant 0$. However, in a structural system composed by several structural components with several possible failure modes associated, the system failure event is generally given by a complex combination of failure modes. For a basic system with $m$ failure modes in series, the system failure probability is defined by:

$$p_f = \mathrm{P}\left[\bigcup_{i=1}^{m}(M_i \leqslant 0)\right] \tag{2}$$

while for the parallel case it is,

$$p_f = \mathrm{P}\left[\bigcap_{i=1}^{m}(M_i \leqslant 0)\right]. \tag{3}$$

These are the elementary cases considered in structural systems reliability analysis [4]. However, a complex combination of series and parallel systems of failure modes may be needed to define the system failure event. In general, a series system of parallel subsystems of failure modes can be adopted.

As mentioned in the introduction, computation of the failure probability of a system of failure modes is in general a difficult task, due to limitations on the approximate analytical methods or the excessive computational cost that may be involved when numerical simulation methods as the Monte Carlo are adopted. The new Monte Carlo based simulation method proposed in [1] allows the estimation of the system failure probability with low to moderate computational cost. With this method the system safety margins are formulated in the following way:

$$M_i(\lambda) = M_i - \mu_i(1-\lambda) \tag{4}$$

where $M_i$ is a system safety margin, given by Eq. (1), and $\mu_i = E\,[M_i]$ is the mean value of $M_i$. $\mu_i$ is generally unknown a priori, but it is estimated with high accuracy as part of the proposed method. The parameter $\lambda$ can assume values in the interval $0 \leqslant \lambda \leqslant 1$ and its effect on the system failure probability may be interpreted as a scale factor. The original system is obtained for $\lambda = 1$, and for $\lambda = 0$ the system is highly prone to failure, as the mean value of the system safety margins is $E\,[M_i(0)] = 0$. For small to intermediate values of $\lambda$ the increase in the system failure probability is sufficiently high to get accurate estimates of the failure probability by crude Monte Carlo simulation with moderate computational cost. Using the modified system safety margins, the failure probability as a function of $\lambda$ for the series and parallel cases given by Eqs. (2) and (3) can be written as:

$$p_f(\lambda) = \mathrm{P}\left[\bigcup_{i=1}^{m}(M_i(\lambda) \leqslant 0)\right] \tag{5}$$

for the series system and,

$$p_f(\lambda) = \mathrm{P}\left[\bigcap_{i=1}^{m}(M_i(\lambda) \leqslant 0)\right] \tag{6}$$

for the parallel system. For more complex combinations of failure modes similar expressions can be defined, using Eq. (4) to describe the system safety margins as a function of $\lambda$. Analyzing the behavior of $p_f(\lambda)$ we may conclude that this function decreases monotonically from a high value at $\lambda = 0$ to a typically small target value at $\lambda = 1$. It was shown in [1] that $p_f(\lambda)$ can be approximated by a parametric function of the form:

$$p_f(\lambda) \underset{\lambda \to 1}{\approx} q(\lambda)\exp\{-a(\lambda-b)^c\} \tag{7}$$

where $q(\lambda)$ is a slowly varying function compared with the exponential function $\exp\{-a(\lambda-b)^c\}$ for values of $\lambda$ close to 1. For practical applications it was concluded that Eq. (7) can be simplified by considering $q(\lambda) = q$, with $q$ a constant or additional parameter. Hence, the approximating function for $p_f(\lambda)$ is implemented in the following form:

$$p_f(\lambda) \approx q\exp\{-a(\lambda-b)^c\} \quad \text{with } \lambda_0 \leqslant \lambda \leqslant 1 \tag{8}$$

for a suitable value of $\lambda_0$, which is defined as a tail marker. An important part of the method is therefore to identify a suitable $\lambda_0$ to ensure that the right hand side of Eq. (8) represents a good approximation of $p_f(\lambda)$ in the interval $\lambda_0 \leqslant \lambda \leqslant 1$. A suitable initial value for the tail marker is $\lambda_0 = 0.3$, however slightly different values can be considered. The practical importance of the approximation provided by Eq. (7) is that the target failure probability, defined as $p_f = p_f(1)$, can be obtained from values of $p_f(\lambda)$ for $\lambda < 1$. This is the main concept of the Monte Carlo based reliability estimation method proposed in [1], as it is easier to estimate the failure