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Vulnerability-based robust design optimization of imperfect shell structures

Vissarion Papadopoulos *, Nikos D. Lagaros

Institute of Structural Analysis and Seismic Research, National Technical University of Athens, Athens 15780, Greece

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ABSTRACT

A stochastic vulnerability-based robust design procedure of isotropic shell structures possessing uncertain initial geometric as well as material and thickness properties that are modeled as random fields is assessed against conventional and reliability-based robust design procedures. The main idea of the vulnerability-based design philosophy is to achieve robust optimum designs while allowing designers to determine explicitly accepted probabilities that various performance objectives will not be exceeded, by introducing additional probabilistic (vulnerability) constraints. For this purpose, a stochastic finite element methodology is incorporated into the framework of an efficient two-objective robust design optimization formulation. This combined approach is then implemented in order to obtain optimum designs of an ‘‘imperfect" shell structure involving random geometric deviations from its perfect geometry as well as a spatial variability of its modulus of elasticity and thickness. Two-objective functions, the material volume of the structure and the coefficient of variation of the buckling load of the shell, are used for the description of the optimization problem, subject to deterministic, reliability and vulnerability constraints.

1. Introduction

Deterministic formulations of structural optimization problems are not capable to reach unbiased, feasible and realistic optimum structural designs due to the fact that such formulations ignore the uncertainties involved in the various parameters affecting the structural behaviour. Once a deterministic optimum design is materialized to a real physical system, its optimal performance may vanish because of the unavoidable scattering values of the parameters, which might also be unfavorable since the performance of the "implemented" design may be far worse than expected. In practical applications, however, finding the global optimum in the presence of uncertainties in various structural parameters, such as material properties, geometric imperfections, loading variations, uncertain boundary conditions, etc., is a difficult and computationally intensive task, since for any candidate design a full stochastic analysis has to be performed for estimating various statistical quantities. Efficient methodologies are therefore required for the solution of the stochastic and the optimization part of the problem. A complete survey on these methodologies can be found in [1].

Such probabilistic optimum design formulations are usually distinguished, depending on the probabilistic system response quantities that are taken into account, in two categories: reliability-based optimization (RBO) [2–4] and robust design optimization (RDO) [5–7]. The main goal of RBO formulations is to design for safety with respect to extreme events by determining design points that are located within a range of target failure probabilities. On the other the fundamental principle of RDO is to improve product quality or stabilize performances by minimizing the effects of variations without eliminating their causes. This is usually achieved by considering the mean value and/or the standard deviation of a response quantity as an objective function and trying to establish the designs that minimize the aforementioned quantities considering deterministic or reliability constraints. Further to RDO and RBO formulations the reliability-based robust design optimization (RRDO) formulation has been addressed [8] in order to account for the influence of the probabilistic constraints in the framework of RDO of realistic structures. The recent advances in RBO, RDO and RRDO structural optimization problems can be found in the book by Tsompanakis et al. [9].

The traditional design procedures followed for imperfect shell-type structures are based on conservative corrections of deterministic non-linear analyses by means of the well-known empirical "knock-down" factors. A step forward to the aforementioned "traditional" procedure was recently achieved through accurate predictions of the scatter of the buckling loads that was accomplished via realistic descriptions of the various uncertainties involved in the problem. Such task is realizable only in the framework of stochastic finite element method (SFEM) formulations that can efficiently and accurately handle the geometric as well as physical non-linearities of shell-type structures [10–16]. This type of SFEM approaches, however, can provide with reasonable

* Corresponding author.
E-mail addresses: vpapado@central.ntua.gr (V. Papadopoulos), nlagaros@central.ntua.gr (N.D. Lagaros).

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estimates of the scatter of the buckling loads only if the full probabilistic characteristics (marginal pdfs and correlation structures) of the involved stochastic fields are derived on the basis of corresponding experimental surveys. As this requirement is rarely satisfied, such SFEM approaches are usually implemented as "worst case" studies, based on sensitivity analyses with respect to the aforementioned parameters.

A design procedure is proposed in the present paper that addresses the vulnerability-based optimization (VBO) concept of isotropic shell structures possessing uncertain initial geometric as well as material and thickness properties that are modeled as random fields. The VBO concept used in this work is based on the trading-off performance and robustness framework that has been proposed by Mourelatos and Liang [17]. The main motivation for proposing the VBO formulation is that despite the fact that RBO and RRDO formulations lead to design points that are located within a range of target failure probabilities, intermediate (prior-to-failure) limit states possibly crucial for the structural behaviour and operational integrity are ignored. The main difference between VBO and RBO formulations is that multiple limit states are considered in VBO, apart from the failure one. Thus, VBO can be considered as an RBO procedure with multiple probabilistic constrains. The main novelty of the proposed procedure with respect to an RBO formulation with multiple probabilistic constrains, is that the definition of the multiple probabilistic constrains, named hereafter "vulnerability constrains", is associated with the classical structural vulnerability analysis. Thus, the vulnerability constraints are related to acceptable damage and/or serviceability limit states, prior and up to total structural failure, of increasing intensity.

In the present paper, the aforementioned VBO design methodology is combined with the robust design optimization leading to the vulnerability-based robust design optimization (VRDO) formulation. In VRDO, an optimum robust design is achieved while the vulnerability (probabilistic) constraints are simultaneously satisfied. This is accomplished by combining a classical vulnerability analysis in the context of a stochastic finite element method (SFEM) approach with an efficient objective robust design optimization formulation. Two-objective functions, the material volume of the structure and the coefficient of variation of the buckling load of the shell, are simultaneously used for the description of the optimization problem, while in addition to deterministic constraints, imposed by the Eurocode 3 [18], vulnerability constraints are also taken into account.

This combined approach is then implemented in order to obtain rational optimum designs of an "imperfect" isotropic shell structure involving stochastic geometric deviations from its perfect geometry as well as a spatial variability of the modulus of elasticity and the thickness of the shell. Two algorithms are employed for the solution of the two-objective optimization problem at hand; the first one is the non-domination sort evolution strategies II (NSES-II) algorithm, which is based on [19], while the second one is the Strength Pareto Evolution Strategies 2 (SPES 2) which is a variant algorithm of the one proposed in [20]. Numerical results are presented for a cylindrical panel, demonstrating the applicability of the proposed stochastic optimization methodology and the improved efficiency of the SPES 2 over the NSES-II algorithm in obtaining rational optimum designs of imperfect shell-type structures, which also satisfy the vulnerability design criteria.

2. Stochastic finite element formulation

2.1. Description of random geometric imperfections

Following an approach similar to the one described in [10], initial geometric imperfections are modeled as 2D-1V homogeneous Gaussian stochastic fields. Thus, geometric imperfections are introduced as fluctuations around a, so called, 'perfect' structural geometry as follows:

$$D(x,y) = D_0(1 + f_1(x,y)) \tag{1}$$

where D_0 is the domain of the perfect shell geometry which in this case coincides with the mean geometry of the structure and $f_1(x,y)$ is a zero mean 2D Gaussian homogeneous stochastic field. The amplitude of the imperfections is controlled by the standard deviation of the stochastic field. The coordinates x, y are the global Cartesian coordinates of the unfolded panel. Moreover, the shape of the imperfections is controlled by the correlation lengths of the stochastic field $f_1(x,y)$ in directions x and y, respectively.

It must be mentioned here that the assumption of homogeneity, although not generally applicable for the description of initial imperfections of shells, is adopted in this study and elsewhere [10,11,14] due to the fact that there is no experimental data available for this particular type of cylindrical panels. However, the proposed approach can be easily extended to non-homogeneous cases by using the spectral representation method together with an evolutionary power spectrum [11] or some other representation method such as the Karhunen–Loeve expansion [16,21]. In cases where there is lack of experimental data, a sensitivity analysis is always required with respect to various probabilistic quantities of the stochastic fields that describe the imperefections [14,16].

2.2. Non-linear finite element analysis

The finite element simulation is performed using the non-linear multilayer triangular shell element TRIC, which is based on the natural mode method. The TRIC shear-deformable facet shell element is a reliable and cost-effective element suitable for linear and non-linear analysis of thin and moderately thick isotropic as well as composite plate and shell structures. The element has 18 degrees of freedom (six per node) and hence 12 natural straining modes. Three natural axial strains and natural transverse shear strains are measured parallel to the edges of the triangle. The natural stiffness matrix is derived from the statement of variation of the strain energy with respect to the natural coordinates. The geometric stiffness is based on large deflections but small strains. The elastoplastic stiffness of the element is obtained by summing up the natural elastoplastic stiffnesses of the element layers.

The solution of the non-linear system of equations at each Monte Carlo simulation is performed using the standard incremental-iterative Newton Raphson algorithm in conjunction with the arc-length path-following technique. The effective stress for the elastoplastic analysis is computed according to the Von Mises yield criterion. Such a procedure enables the prediction of the full non-linear pre and post-buckling load–displacement path. The predicted critical buckling load is assumed to correspond to the load level at which the first negative eigenvalue of the tangent stiffness matrix of the structure appears. A detailed description of the linear elastic, geometric and elastoplastic stiffness matrix of the TRIC shell element can be found in [22–24].

2.3. Stochastic stiffness matrix

For the calculation of the stochastic stiffness matrix of TRIC the modulus of elasticity as well as the thickness of the structure are also considered in the present study as 'imperfections', in addition to geometric imperfections, due to their spatial variability. These parameters are also described as two independent 2D-1V homogeneous stochastic fields

$$\mathrm{E}(x,y) = E_0[1 + f_2(x,y)] \tag{2}$$

$$t(x,y) = t_0[1 + f_3(x,y)] \tag{3}$$