Structural Safety 33 (2011) 196–205

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

On non-probabilistic reliability-based design optimization of structures with uncertain-but-bounded parameters

Zhan Kang [a,*], Yangjun Luo [b], Alex Li [c]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China
[b] School of Mechanics, Civil Engineering & Architecture, Northwestern Polytechnical University, Xi'an 710072, China
[c] Laboratoire de Génie Civil, GRESPI, Université de Reims Champagne-Ardenne, Rue des Crayères, BP1035, 51687 Reims Cedex, France

ARTICLE INFO

Article history:
Received 28 April 2009
Received in revised form 1 March 2011
Accepted 1 March 2011
Available online 26 March 2011

Keywords:
Structural optimization
Uncertainty
Convex model
Non-probabilistic reliability
Sensitivity analysis

ABSTRACT

This paper investigates the formulation and numerical solution of reliability-based optimization of structures exhibiting grouped uncertain-but-bounded variations. Based on the multi-ellipsoid convex model description for grouped uncertain-but-bounded parameters, the mathematical definition of a non-probabilistic reliability index is presented for quantified measure of the safety margin. The optimal design is then formulated as a nested optimization problem. A method based on concerned performance is proposed for regularization of the reliability index constraints. The expensive computation of the non-probabilistic reliability index and its derivative is thus avoided. Numerical examples are given to illustrate the validity and efficiency of the present method.

1. Introduction

Structural optimization techniques have been widely applied to the design of engineering structures. Deterministic structural optimization searches for minimum cost, without considering the uncertainties in design, manufacturing and operating processes. However, in practical engineering, the structural performance always exhibits some degree of variations due to uncertainties of material properties, loading conditions, geometric dimensions, etc. Hence, a proper design procedure must reasonably account for the inherent uncertain nature of a structural system [1]. Non-deterministic structural optimization, including Reliability-Based Design Optimization (RBDO) [2,3] and structural robust design optimization [4–6], is therefore attracting increasing attentions both in theoretical research and practical applications. While robust design aims at minimizing variation of the objective function, RBDO puts more emphasis on reliability of the constraints. In most cases, RBDO is an effective approach to avoid structural failure and to enhance safety in the presence of uncertain parameters.

Reliability-Based Design Optimization has been intensively studied both in the methodology and in applications [7–13]. The analysis of the probabilistic reliability requires precise information on the distribution of the uncertainties involved in the design. However, these data are hardly available in some practical engineering applications where there are only a limited number of samples. Moreover, the probabilistic reliability may be sensitive to the description of the random parameters and thus just small errors in the inputs may yield misleading results [14]. Studies on the construction of probabilistic models on the basis of incomplete information can be dated back to five decades ago. A long tradition in probability theory is to use the maximum entropy approach for setting up target distributions in the absence of sufficient sample data [15,16]. The basic assumption of this approach is that the distribution of maximum entropy under suitable constraints consistent with the available information is the least-biased estimation of the real distribution. The maximum entropy approach tends to produce a distribution that is closest to uniform. It has been successfully used in many practical problems [17,18]. Up to now, the quantification of various uncertainties in realistic systems remains a challenge problem [19] and a number of attempts have been made to apply non-probabilistic models, such as convex model and interval set, for mathematical description of the uncertainty in non-deterministic structural analysis and optimization problems with limited uncertainty information [20].

In many circumstances, the bounds of the uncertainties, compared with the precise probability distribution data, are more easily available. The convex model, which provides an objective description of the boundary of the parameter variations without considering the inner distribution, is suitable to treat those uncertain-but-bounded parameters in the optimization of uncertain structures. In the 1990s, Ben-Haim [21,22] and Elishakoff [23] first discussed the concept of non-probabilistic reliability based on the

* Corresponding author. Tel.: +86 41184706067; fax: +86 41184708932.
E-mail address: zhankang@dlut.edu.cn (Z. Kang).

0167-4730/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.strusafe.2011.03.002

convex model theory. The principal assumption behind this concept is that: a structural system is considered to be more reliable when it allows for a greater extent of uncertainties, and vice versa. In this context, the reliability of a structural system is measured with the maximum extent of uncertainties it permits. Following this idea, researchers have developed a variety of formulations and numerical techniques for implementing the non-probabilistic design optimization, which serves as an alternative to the well-established RBDO approaches when complete information on the uncertainty distribution is not available. Qiu and Elishakoff [24,25], Elishakoff et al. [26] studied the optimal design of truss structures with uncertain-but-bounded parameters using the interval set modelling. The interval set was also employed by Tabakov and Walker [27] to model the manufacturing tolerances in the ply angle optimization of a laminated composite shell using the GA algorithm. Lombardi and Haftka [28], Pantelides and Ganzerli [29] applied the anti-optimization technique to the optimization of structures but only uncertain loading conditions are considered. In a two-level approach relying on anti-optimization, a nested inner-loop optimization problem needs to be solved at each step of the structural optimization. Ganzerli and Pantelides [30] proposed a convex model superposition method for the optimal design of linear structural systems subject to bounded uncertain loads. Making use of the linearity property, they expressed the convex sets of the structural responses (displacements or inner forces) as a linear combination of the effect of each load applied separately on the structure. Thus the lower-level optimization process can be eliminated entirely and the computational cost is greatly reduced.

While the aforementioned convex model-based approaches term a structure as either safe or unsafe, quantified measures of structural reliability are introduced by some studies into optimal design of uncertain-but-bounded structures. Cao and Duan [31] suggested a non-deterministic optimization approach based on non-probabilistic reliability. However, the considered problem is very difficult to be solved though a sequential linearization technique can be applied. Au et al. [32] presented an interesting discussion on the shortcoming of direct use of the maximum allowable value of each individual uncertain parameter as a reliability measure when multiple uncertain variables are concerned. They therefore suggested a so-called unsatisfactory degree to measure the reliability of structures with interval uncertainties. However, the six preference ranges used to describe the unsatisfactory degrees in their approach are rather subjective choices. The similar concept was also used by Jiang et al. [33] in the optimal design of uncertain structures. In a recent paper [34], the authors of this paper presented the topology optimization of continuum structures under reliability constraints of structural deformation in the context of multi-ellipsoid convex model description. It is shown that the proposed approach may yield different structural layouts satisfying various safety margin requirements. Despite of these efforts, more general formulations of structural optimization problems subject to quantified reliability constraints in the framework of multi-ellipsoid convex modelling, as well as associated numerical techniques that enable real-scale applications, have not been adequately studied so far.

This paper aims to investigate the structural optimization problem for non-probabilistic reliability of general behaviours in presence of grouped uncertain-but-bounded parameters. As an adaptation of the conventional probabilistic reliability index definition [35], a new mathematical definition of non-probabilistic reliability index is suggested for objectively measuring the structural safety margin in the context of multi-ellipsoid convex modelling of grouped uncertain-but-bounded parameters. A mathematical model for structural optimization problem considering constraints on such reliability indices is then proposed. It is pointed out that the structural optimization problem using this concept is in nature a nested one and the inner loop takes the form of a Min–max type optimization problem, which usually leads to poor convergence when a direct solution for the constrained optimization problem is sought. This consequently calls for a special regularization strategy. Inspired by the Performance Measure approach (PMA) [12] for the reliability-based design optimization incorporating stochastic uncertainties, we propose a concerned performance-based approach to overcome these difficulties. In this approach, the constraints on the reliability indices are replaced by constraints on the concerned performance and thus the original nested optimization problem is reformulated into one that can be more easily handled. Compared with the reliability-index-based approach, the design sensitivity analysis for the constraint condition is more straightforward and the iteration stability can be notably improved since the reformulated problem is an inherently more robust one. The present method can be employed to find optimum design of structures with uncertainties in material properties, geometrical dimensions and loading conditions. The novelty of the proposed method lies in the definition of non-probabilistic reliability index in the context of multi-ellipsoid modelling and the use of concerned performance approach in solving the optimization problem incorporating such reliability requirements. Finally, several examples, with regard to optimal designs of two truss structures and a wing box structure, are given to demonstrate the applicability and effectiveness of the present method.

2. Multi-ellipsoid convex modelling for grouped uncertainties

Several frequently used convex models are the envelope-bound convex model, the instantaneous energy-bound convex model, the cumulative energy-bound convex model and the ellipsoid convex model [25]. In this study, a multi-ellipsoid convex model is applied for the uncertainty description.

Assume the uncertain parameters are collected in the vector $\mathbf{x} \in R^n$. The vector $\mathbf{x}$ is first transformed into a dimensionless vector $\boldsymbol{\delta} \in R^n$ and the components of the vectors $\mathbf{x}$ and $\boldsymbol{\delta}$ are related by

$$x_i = (1 + \delta_i) \cdot \bar{x}_i \quad (i = 1, 2, \ldots, n), \tag{1}$$

where $\bar{x}_i$ denotes the nominal value of the ith uncertain parameter.

In the ellipsoid convex model, all the possible values of uncertain parameters are assumed to be bounded by a multidimensional (hyper-) ellipsoid, which is expressed by

$$\boldsymbol{\delta} \in \mathbb{E}(\mathbf{W}, \varepsilon) = \{\boldsymbol{\delta} : \boldsymbol{\delta}^T \mathbf{W} \boldsymbol{\delta} \leqslant \varepsilon^2\}, \tag{2}$$

where $\mathbf{W}$ is the characteristic matrix of the ellipsoid, and it is a symmetric positive-definite real matrix defining the orientation and aspect ratio of the ellipsoid, while ε is a real number defining the magnitude of the parameter variability. Here, the orientation of the ellipsoid reflects the correlations between the uncertainties. The matrix $\mathbf{W}$ and the number ε can be objectively determined through outputs of instrument measurements.

In many structural design problems, the uncertainties exhibited by a structural system may arise from various sources. It might be more realistic to assume that variations of parameters from different sources are uncorrelated, which means, the uncertainties can be divided into groups and the variation bounds of a parameter depend only upon other uncertain parameters of the same group. In such circumstances, it is reasonable to treat each group of uncertainties with an individual ellipsoid convex set. Such an uncertainty description is referred to as the *multi-ellipsoid convex model* [34]. Supposing k ellipsoid sets are employed, we express the vector of the grouped uncertain parameters by

$$\mathbf{x}^{\mathrm{T}} = \{\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}, \ldots, \mathbf{x}_k^{\mathrm{T}}\}, \tag{3}$$