Structural Safety 32 (2010) 293–304

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Discrete–continuous variable structural optimization of systems under stochastic loading

Hector A. Jensen [a,*], Michael Beer [b]

[a] Department of Civil Engineering, Santa Maria University, Casilla 110-V, Valparaiso, Chile
[b] Department of Civil Engineering, National University of Singapore, Singapore 117576, Singapore

ARTICLE INFO

Keywords:
Approximation concepts
Dual formulation
Non-linear systems
Reliability-based optimization
Sensitivity analysis

ABSTRACT

The paper deals with the optimization of structural systems involving discrete and continuous sizing type of design variables. In particular, the reliability-based optimization of non-linear systems subject to stochastic excitation where some or all of the design variables are discrete is considered. The reliability-based optimization problem is formulated as the minimization of an objective function subject to multiple reliability constraints. The probability that design conditions are satisfied within a given time interval is used as measure of system reliability. The basic mathematical programming statement of the structural optimization problem is converted into a sequence of explicit approximate primal problems of separable form. The explicit approximate primal problems are solved by constructing continuous explicit dual functions, which are maximized subject to simple non-negativity constraints on the dual variables. A gradient projection type of algorithm is used to find the solution of each dual problem. The effectiveness of the method is demonstrated by presenting a numerical example of a non-linear system subject to stochastic ground acceleration.

1. Introduction

For many structural optimization problems the design variables must be selected from a list of discrete values. For example, cross-sectional areas of truss members have to be chosen from a list of commercially available member sizes. Also, the growing use of fiber composite materials in different type of structures underlines the importance of being able to treat structural optimization problems where some or even all of the design variables are discrete. Thus, due to manufacturing limitations the design variables cannot be considered as continuous but should be treated as discrete in a large number of practical design situations. In the reliability-based optimization literature that deals with stochastic dynamical systems little attention has been given to dealing with discrete design variables. In fact, the optimal design of structural systems under stochastic like loadings such as seismic excitations, water wave excitations, wind excitations, traffic loadings, etc., is usually carried out by considering continuous design variables. In this work attention is directed to reliability-based optimization problems of non-linear stochastic dynamical systems involving discrete and continuous sizing type of design variables. The reliability-based optimization problem is formulated as the minimization of an objective function subject to multiple reliability constraints. The probability that design conditions are satisfied within a given time interval is used as measure of system reliability. Standard methods attack the discrete variable design optimization problem by employing discrete or integer variable algorithms to treat the problem directly in the primal variable space (branch and bound techniques, combinatorial methods, evolution-based optimization techniques, etc.) [9,18,27,30]. These methods are generally applicable and some of them have been used as efficient tools for discrete variable structural optimization considering uncertainties [19,21]. However, these approaches are associated with high computational costs when dealing with stochastic dynamical systems since a considerable number of reliability analysis calls are required during the optimization process. An alternative approach for solving structural synthesis problems involving a mix of discrete and continuous sizing type of design variables is investigated in this paper. It is based on the combined use of approximation concepts and dual methods. The basic mathematical programming statement of the structural optimization problem is converted into a sequence of explicit approximate primal problems of separable form. For this purpose, the objective function and the reliability constraints are approximated by using a hybrid form of linear and reciprocal approximations. Specifically, a conservative approximation is considered in the present formulation [12,24]. The approximations are combined with an efficient simulation technique to generate explicit expressions of the reliability constraints in terms of the continuous and discrete design variables. In principle, the explicit

* Corresponding author.
E-mail addresses: hector.jensen@usm.cl (H.A. Jensen), cvebm@nus.sg (M. Beer).

0167-4730/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.strusafe.2010.03.007

approximate primal problems can be solved by any optimization algorithm that treats continuous and discrete design variables. In the present implementation they are solved by constructing continuous explicit dual functions which are maximized subject to simple non-negativity constraints on the dual variables. The evaluation of the dual function is direct since it requires the minimization of a series of one-dimensional minimization problems. A gradient projection type of algorithm is used to find the solution of each dual problem. In general the proposed optimization scheme converges in few iterations.

The structure of the paper is as follows. In Section 2 the general formulation of the problem is presented. Sections 3 and 4 review some hybrid forms of linear and reciprocal approximations in the context of sequential approximate optimization. Next, Section 5 presents the corresponding dual formulation of the approximate problems. The application of the general method to reliability-based optimization of structural systems under stochastic loading, which is the main scope of the paper, is considered in Section 6. Several implementation aspects are discussed in detail. Finally, a numerical example is presented in Section 7.

2. Formulation

Consider a general structural optimization problem defined as the identification of a vector $\{x\}$ of design variables to minimize an objective function, that is

$$\text{Minimize} \quad F(\{x\})$$

$$\text{subject to design constraints} \quad h_j(\{x\}) \leqslant 0, \quad j = 1, \ldots, n_c \tag{1}$$

with side constraints

$$x_i^l \leqslant x_i \leqslant x_i^u, \quad i \in I_C \tag{2}$$

and

$$x_i \in X_i = \{\bar{x}_i^l, l = 1, \ldots, n_i\}, \quad i \in I_D \tag{3}$$

where I_C denotes the set of indices for continuous design variables while I_D denotes the set of indices for discrete design variables. The x_i^l and x_i^u denote the lower and upper limits for the design variables that are continuous i.e. $i \in I_C$, and Eq. (3) represents the side constraints for the design variables that are discrete i.e. $i \in I_D$. The set X_i represents the available discrete values for the design variable $x_i, i \in I_D$, listed in ascending order.

3. Approximation concepts

The solution of the structural optimization problem given by Eqs. (1)–(3) is obtained by transforming it into a sequence of sub-optimization problems having a simple explicit algebraic structure. For this purpose, the objective and the constraint functions are approximated by using a hybrid form of linear and reciprocal approximations. In particular, a conservative approximation is considered in the present formulation [8,12–14,24]. Let $p(\{x\})$ be a generic performance function, i.e. the objective or constraint functions, and $\{x^0\}$ a point in the design space. The function $p(\{x\})$ is approximated about the point $\{x^0\}$ as

$$p(\{x\}) \approx \tilde{p}(\{x\}) = p(\{x^0\}) + \sum_{(+)} \frac{\partial p(\{x^0\})}{\partial x_i}(x_i - x_i^0) + \sum_{(-)} \frac{\partial p(\{x^0\})}{\partial x_i} \frac{x_i^0}{x_i}(x_i - x_i^0) \tag{4}$$

where $\sum_{(+)}$ and $\sum_{(-)}$ mean summation over the variables belonging to group (+) and (−), respectively. Group (+) contains the variables for which $\partial p/\partial x_i(\{x^0\})$ is positive, and group (−) includes the remaining variables. The expansion given by Eq. (4) corresponds to a linear approximation in terms of the direct variables (x_i) for the variables belonging to group (+), and a linear approximation in terms of the reciprocal variables $(1/x_i)$ for the variables belonging to group (−). An attractive property of this mixed linearization, called convex linearization, is that it yields the most conservative approximation among all possible combinations of direct/reciprocal variables (see Appendix A). In addition, the conservative approximation is a convex and separable function. The above properties, that is, convexity and separability are important since they are the basis of the optimization scheme proposed in the sequel.

4. Approximate primal problem

Applying the above linearization approach to the quantities involved in the optimization problem (1)–(3) yields the following approximate primal problem

$$\begin{aligned} &\text{Minimize} \quad \sum_{(i^+)} \frac{\partial F(\{x^0\})}{\partial x_i} x_i - \sum_{(i^-)} \frac{\partial F(\{x^0\})}{\partial x_i} \frac{(x_i^0)^2}{x_i} \\ &\text{subject to} \quad \sum_{(i_j^+)} \frac{\partial h_j(\{x^0\})}{\partial x_i} x_i - \sum_{(i_j^-)} \frac{\partial h_j(\{x^0\})}{\partial x_i} \frac{(x_i^0)^2}{x_i} \leqslant \bar{h}_j \\ &\qquad j = 1, \ldots, n_c \end{aligned} \tag{5}$$

where

$$\bar{h}_j = \sum_{(i_j^+)} \frac{\partial h_j(\{x^0\})}{\partial x_i} x_i^0 - \sum_{(i_j^-)} \frac{\partial h_j(\{x^0\})}{\partial x_i} x_i^0 - h_j(\{x^0\}) \tag{6}$$

with side constraints

$$x_i^l \leqslant x_i \leqslant x_i^u \quad i \in I_C \tag{7}$$

and

$$x_i \in X_i = \{\bar{x}_i^l, l = 1, \ldots, n_i\}, \quad i \in I_D \tag{8}$$

where the groups (i^+) and (i_j^+) contain the variables for which the partial derivatives of the objective function and constraint functions are positives, respectively, and the groups (i^-) and (i_j^-) include the remaining variables. As before I_C denotes the set of indices for continuous design variables, I_D denotes the set of indices for discrete design variables, and X_i represents the set of available discrete values for the design variable $x_i, i \in I_D$, listed in ascending order. It is noted that for the purpose of constructing the approximations of the objective and constraint functions all variables are assumed to be continuous. From the optimization point of view, however, only the design variables $x_i, i \in I_C$ are continuous. An optimization scheme that takes advantage of the particular structure of the optimization problem is considered for the solution of the approximate primal problem. The approximate primal optimization problem is convex for the pure continuous variable case. This is due to the fact that the approximations of the objective and constraint functions are convex and therefore the feasible domain of the approximate primal optimization problem is convex. Convex problems are guaranteed to have only a single optimum, and they are amenable to treatment by dual methods [12]. In addition, the resulting approximate primal problem is separable since both the objective and the constraints are separable functions. Due to the convexity and separability of the convex linearization, the explicit primal problem can be efficiently solved by dual methods of mathematical programming. For the general case of mixed variables, where some of the design variables are discrete and the others are continuous, the extension of the dual formulation is not rigorous from a strict mathematical point of view because the approximate primal problem (5)–(8) is no longer convex. However,