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Extreme value statistics of combined load effect processes

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ABSTRACT

The design of structures subjected to environmental loads, e.g. offshore platforms, ships, aircrafts, tall buildings, would usually include analysis of combinations of load effects, for example, the combination of stress components in structural joints. There are various yield criteria describing boundaries of structural element failure. This paper focuses on the development of efficient and accurate methods for estimating extreme response statistics of combined load effect processes. This latter issue is crucial for reliability design.

1. Introduction

A prominent problem in the design of structures subjected to random loads is to find methods for the combination of resulting load effects at high and extreme response levels. In codified design this is usually implemented as linear combination rules of specified characteristic values of the individual load effects [1,2]. For nonlinear dynamic structures, the precision level of such procedures would seem highly questionable. One of the reasons for adopting such simplified procedures is the complexity of the task of accurately predicting the extreme value statistics of the combined load effects, even in the case of linear combinations. Over the years, several simplified procedures have been suggested for the linear combination of load effects, most notably the Ferry Borges–Castanheta method [3], Turkstra's rule [1,4], the load coincidence method [2,5], the square root of sum of squares (SRSS) method [5,6], and the point crossing approximation method [1,7]. An important shortcoming of these combination procedures is that they apply mainly to the case of independent load effect components. An effort to extend Turkstra's rule to dependent processes is described in [8].

The authors have developed an accurate and efficient method for estimation of extreme values of stochastic processes [9–11]. The method is based on Monte Carlo simulation. It is therefore eminently suitable for use in the estimation of extreme values of combined stochastic load effect models since Monte Carlo simulation is very often possible in such cases. In this paper we shall illustrate the usefulness and accuracy of the estimation method for two specific load combination problems.

2. Combination of stochastic load effects

The general formulation of the load effect combination problem to be studied in this paper is the following,

$$H(t) = h[X_1(t), .., X_N(t)], \tag{1}$$

where the stochastic load effect component processes $X_1(t), \ldots, X_N(t)$ are combined according to a specified deterministic function h to produce the load effect combination process $H(t)$. The component processes may, e.g. derive from a vector solution process of a dynamic model for the structural response of an offshore platform to random waves. They may often be modelled as stationary stochastic processes, but that is not a requirement for the application of the methods developed in this paper.

The typical problem to answer concerning the load effect combination process $H(t)$ is to determine the probability of exceeding a critical threshold h_c during a specified time interval T. Let us call this the failure probability and denote it by $p_f = p_f(T)$. Hence, the goal will be to find

$$p_f = 1 - \text{Prob}[H(t) < h_c;\ 0 \leqslant t \leqslant T]. \tag{2}$$

In many practical applications, the structure of the process $H(t)$ is quite involved and the dimension N can be relatively high. This makes a direct analytical approach virtually impossible in general. In such cases, Monte Carlo simulations of some sort would seem to be the most attractive way to provide estimates of the failure probability. The purpose of this study is to describe a simple Monte

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Carlo technique, vastly more efficient than brute force standard Monte Carlo simulations. A special procedure for estimating the extremes of $H(t)$ will be described and its performance demonstrated.

3. Extreme value prediction

Let $N^+(\xi; t_1, t_2)$ denote the random number of times that the process $H(t)$ upcrosses the level ξ during the time interval (t_1, t_2). Assuming stationarity, $E[N^+(\xi; t_1, t_2)] = E[N^+(\xi; 0, 1)] \cdot (t_2 - t_1)$, where $E[N^+(\xi; 0, 1)]$ is referred to as the mean rate of ξ-upcrossings of $H(t)$. We shall use the notation $\nu^+(\xi) = E[N^+(\xi; 0, 1)]$.

Let $M(T) = \max\{H(t) : 0 \leqslant t \leqslant T\}$ denote the extreme value of the process $H(t)$ from Eq. (2) over the time interval of length T. The cumulative distribution function of $M(T)$ under the Poisson assumption is given in terms of the mean upcrossing rate by the following relation for a stationary short-term loading condition,

$$\mathrm{Prob}(M(T) \leqslant \xi) = \exp\{-\nu^+(\xi)T\}, \tag{3}$$

where $\nu^+(\xi)$ denotes the mean rate of upcrossings of the level ξ by the process $H(t)$. Eq. (3) brings out the crucial role of the mean upcrossing rate in determining the extreme value distribution.

For a long-term situation, which implies a nonstationary response process, Eq. (3) is replaced by

$$\mathrm{Prob}(M(T) \leqslant \xi) = \exp\left\{-\int_0^T \nu^+(\xi; t)\mathrm{d}t\right\}, \tag{4}$$

where $\nu^+(\xi; t)$ denotes the mean level upcrossing rate at time t, and T equals the long term period considered. For practical purposes, this is rewritten as [12]

$$\mathrm{Prob}(M(T) \leqslant \xi) = \exp\{-T\int_w \nu^+(\xi; w) f_W(w)\mathrm{d}w\}, \tag{5}$$

where $f_W(w)$ denotes the long-term (ergodic) PDF of relevant parameters $W = (W_1, \ldots, W_k)$. In the case of, e.g. offshore structures, typically $W = (H_s, T_p)$, where H_s=significant wave height and T_p=spectral peak period, which implies that the long-term PDF can be estimated from the scatter diagram of the sea states at the specified location of the structure under study.

In practice, Eq. (5) would be expressed as

$$\mathrm{Prob}(M(T) \leqslant \xi) \approx \exp\left\{-T\sum_{j=1}^{m} c_j \nu^+(\xi; w^{(j)})\right\}, \tag{6}$$

where a suitable, representative choice of parameter values $w^{(j)}$, $j = 1, \ldots, m$, has been made, and c_j are suitable weight parameters.

In all cases, the approximation of the failure probability is expressed as $p_f = 1 - \mathrm{Prob}(M(T) \leqslant h_f)$.

4. Empirical estimation of the mean upcrossing rate

In the previous section it was shown that the key to providing estimates of the extreme values of the response process $X(t)$ on the basis of simulated response time histories, is the estimation of the mean upcrossing rate. By assuming the requisite ergodic properties of the response process for a short-term condition, the mean upcrossing rate is conveniently estimated from the ergodic mean value. That is, it may be assumed that,

$$\nu^+(\xi) = \lim_{t\to\infty} \frac{1}{t} n^+(\xi; 0, t), \tag{7}$$

where $n^+(\xi; 0, t)$ denotes a realization of $N^+(\xi; 0, t)$, that is, $n^+(\xi; 0, t)$ denotes the counted number of upcrossings during time t from a particular simulated time history for which the starting point $t = 0$ is suitably chosen. In practice, k time histories of a specified length, T_0 say, are simulated. The appropriate ergodic mean value estimate of $\nu^+(\xi)$ is then

$$\hat{\nu}^+(\xi) = \frac{1}{kT_0}\sum_{j=1}^{k} n_j^+(\xi; 0, T_0), \tag{8}$$

where $n_j^+(\xi; 0, T_0)$ denotes the counted number of upcrossings of the level ξ by time history no. j. This will be the approach to the estimation of the mean upcrossing rate adopted in this paper.

For a suitable number k, e.g. $k \geqslant 20$, and provided that T_0 is sufficiently large, a fair approximation of the 95 confidence interval for the value $\nu^+(\xi)$ can be obtained as $\mathrm{CI}_{0.95}(\xi) = (C^-(\xi), C^+(\xi))$, where

$$C^{\pm}(\xi) = \hat{\nu}^+(\xi) \pm 1.96\frac{\hat{s}(\xi)}{\sqrt{k}}, \tag{9}$$

and the empirical standard deviation $\hat{s}(\xi)$ is given as

$$\hat{s}(\xi)^2 = \frac{1}{k-1}\sum_{j=1}^{k}\left(\frac{n_j^+(\xi; 0, T_0)}{T_0} - \hat{\nu}^+(\xi)\right)^2. \tag{10}$$

Note that k and T_0 may not necessarily be the number and length of the actually simulated response time series. Rather, they may be chosen to optimize the estimate of Eq. (10). If initially, $\tilde{k}$ time series of length $\widetilde{T}$ are simulated, then $k = \tilde{k}k_0$ and $\widetilde{T} = k_0 T_0$. That is, each initial time series of length $\widetilde{T}$ has been divided into k_0 time series of length T_0, assuming, of course, that $\widetilde{T}$ is large enough to allow for this in an acceptable way. The consistency of the estimates obtained by Eq. (10) can be checked for large values of ξ by the observation that $\mathrm{Var}[N^+(\xi; 0, t)] = \nu^+(\xi)t$ since $N^+(\xi; 0, t)$ is then a Poisson random variable by assumption. This leads to the equation

$$\hat{s}(\xi)^2 = \frac{1}{k}\mathrm{Var}\left[\sum_{j=1}^{k}\frac{N_j^+(\xi; 0, T_0)}{T_0}\right] = \frac{\nu^+(\xi)}{T_0}, \tag{11}$$

where $\{N_1^+(\xi; 0, T_0), \ldots, N_k^+(\xi; 0, T_0)\}$ denotes a random sample with a possible outcome $\{n_1^+(\xi; 0, T_0), \ldots, n_k^+(\xi; 0, T_0)\}$. Hence, $\hat{s}(\xi)^2/k \approx \nu^+(\xi)/kT_0$. Since this last relation is consistent with the adopted assumptions, it could have been used as the empirical estimate of the sample variance in the first place. It is also insensitive to the blocking of data discussed above since $kT_0 = \tilde{k}\widetilde{T}$. However, the advantage of Eq. (10) is that it applies whatever the value of ξ, and it does not rely on any specific assumptions about the statistical distributions involved.

Assuming now that we have obtained empirical estimates of the mean upcrossing rate, either for a short-term or a long-term condition, the problem then becomes one of optimal use of the information available. The solution to this problem proposed in the present paper is based on the observation that for most of the dynamic systems met in practical applications, it is possible to make a specific assumption about the behaviour of the mean ξ-upcrossing rate as a function of the level ξ. This is based on the underlying assumption that the appropriate asymptotic extreme value distribution for the response data under study is the Gumbel distribution [13,14]. As will be demonstrated in the next section, the mean upcrossing rate tail, say for $\xi \geqslant \xi_0$, behaves in a manner largely determined by a function of the form $\exp\{-a(\xi - b)^c\}$ ($\xi \geqslant \xi_0$) where a, b and c are suitable constants. Hence, as discussed in detail in [9], it may be assumed that

$$\nu^+(\xi) \approx q(\xi)\exp\{-a(\xi - b)^c\}, \quad \xi \geqslant \xi_0, \tag{12}$$

where the function $q(\xi)$ is slowly varying compared with the exponential function $\exp\{-a(\xi - b)^c\}$ for tail values of ξ. By plotting $\log|\log(\nu^+(\xi)/q(\xi))|$ versus $\log(\xi - b)$, it is expected that an almost perfectly linear tail behaviour will be obtained. Now, as it turns out, the function $q(\xi)$ can be largely considered as a constant for tail values of ξ. This suggests a linear extrapolation strategy obtained by replacing $q(\xi)$ by a suitable constant value, q say.