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Finite element reliability analysis of bridge girders considering moment–shear interaction

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ABSTRACT

Reliability analysis is necessary in bridge design to determine which parameters have the most significant influence on the structural response to applied loadings. To support finite element reliability applications, analytical response sensitivities are derived with respect to uncertain material properties, girder dimensions, reinforcing details, and moving loads by the direct differentiation method (DDM). The resulting expressions have been implemented in the general finite element framework OpenSees which is well suited to the moving load analysis of bridges. Numerical examples verify the DDM response sensitivity equations are correct, then a first-order reliability analysis shows the effect uncertain parameters have on the interaction of negative moment and shear force near the supports of a continuous reinforced concrete bridge girder. A unique contribution is the treatment of moment–shear interaction using Lamé curves with foci calculated from MCFT equations. In addition, the analysis demonstrates non-seismic bridge engineering applications that have been developed in the OpenSees framework.

1. Introduction

The prediction of structural performance and modeling the response of girder members under moving vehicle loads are essential in bridge design. Modeling assumptions and natural randomness in material properties, geometry, and loading make the girder response uncertain. This uncertainty is taken into account by load and resistance factors [1]; however, these aggregate factors do not indicate how the bridge response will change as a function of changes in individual parameters that may be of interest to a designer. Reliability analysis is required to assess the effect parameter variations will have on bridge response and to determine which parameters control the response. Repeated analyses with perturbed parameters lead to the response sensitivity; however, when there is a large number of parameters, this approach can be computationally intense [2].

Several researchers have used reliability methods based on Monte Carlo simulation as an assessment tool for highway bridges [3–5]. First- and second-order reliability methods (FORM and SORM) represent alternative approaches to probabilistic assessment. In these methods, it is necessary to find the most probable failure point by solving a constrained optimization problem. Several algorithms are available to solve such problems and their common characteristic is the need to compute the gradient of the structural response, or response sensitivity, in order to find the failure point. When finite element analysis is used to evaluate the performance function for reliability methods, it is often difficult to implement the software that is necessary to compute gradients of the finite element response.

Most gradient-based finite element software instead rely on finite difference calculations where the analysis is called repeatedly for every realization of the uncertain parameters. In addition to the computational inefficiency of repeated analyses, this approach can lead to inaccurate search directions depending on the size of the parameter perturbations. A more accurate and efficient approach to evaluate gradients in reliability analysis is the direct differentiation method (DDM), which is based on the exact differentiation of the equations that govern the structural response [6]. The response sensitivity equations are implemented alongside the ordinary finite element response equations and are computed at the same precision rate without repeated analyses.

The development of the finite element software framework OpenSees [7] represents one of the first attempts to characterize all major sources of uncertainty in finite element analysis and to compute analytic response sensitivity using an object-oriented approach [8,9]. OpenSees was developed for earthquake engineering applications and several researchers have used the framework to assess the seismic response of bridges. The OpenSees framework is suitable to the repetitive nature of moving load analysis since users build and analyze models via commands added to the fully programmable Tcl scripting language [10]. As a result, OpenSees

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is suited to developing applications for moving load reliability analysis.

The objective of this paper is to use the well-established response sensitivity modules of OpenSees to assess the reliability of bridge girders subjected to moving loads. The presentation begins with a derivation of the sensitivity formulation for material properties, section dimensions, reinforcement details, and moving load parameters in bridge girders. The DDM approach for moving loads is verified by comparison with finite difference calculations and a first-order reliability analysis of bridge girder moment–shear interaction concludes the paper. In the reliability analysis, a third-order Lamé curve whose foci are determined from MCFT equations represents the limit state function for moment–shear interaction.

2. Governing response sensitivity equations

Response sensitivity calculations by the DDM consist of analytical differentiation of the equations that govern the structural response. In this study, the structural response is found by solving the equations of static equilibrium. Impact factors approximate dynamic load effects. The equilibrium equations are described in terms of the vector, $\mathbf{\Theta}$, which contains the uncertain material, geometric and load parameters of a structural model

$$\mathbf{P}_r(\mathbf{U}(\mathbf{\Theta}), \mathbf{\Theta}) = \mathbf{P}_f(\mathbf{\Theta}) \tag{1}$$

The nodal displacement vector, $\mathbf{U}(\mathbf{\Theta})$, depends on the parameters, $\mathbf{\Theta}$, and load history. The resisting force vector, $\mathbf{P}_r$, which is assembled from element contributions by standard finite element procedures, depends on the structural parameters explicitly, as well as implicitly via the nodal displacements. The vector, $\mathbf{P}_f$, contains nodal loads, which also may depend on the parameters in $\mathbf{\Theta}$.

Considering the chain rule of differentiation, the derivative of Eq. (1) with respect to a single parameter, θ, in $\mathbf{\Theta}$, is:

$$\mathbf{K}_T \frac{\partial \mathbf{U}}{\partial \theta} + \left.\frac{\partial \mathbf{P}_r}{\partial \theta}\right|_{\mathbf{U}} = \frac{\partial \mathbf{P}_f}{\partial \theta} \tag{2}$$

where the tangent stiffness matrix, $\mathbf{K}_T = \partial \mathbf{P}_r / \partial \mathbf{U}$, is the partial derivative of the resisting force vector with respect to the nodal displacements. The derivative of the nodal load vector, $\partial \mathbf{P}_f / \partial \theta$, is nonzero only if the parameter, θ, represents a nodal load. The vector, $\partial \mathbf{P}_r / \partial \theta|_{\mathbf{U}}$, is the conditional derivative of the resisting force vector under the condition that the nodal displacements $\mathbf{U}$ are held fixed. This vector is assembled from the conditional derivative of local forces, $\partial \mathbf{q} / \partial \theta|_{\mathbf{v}}$, from each element in the structural model in the same manner as the resisting force vector itself. The nodal response sensitivity is then found by solving the following system of linear equations:

$$\frac{\partial \mathbf{U}}{\partial \theta} = \mathbf{K}_T^{-1}\left(\frac{\partial \mathbf{P}_f}{\partial \theta} - \left.\frac{\partial \mathbf{P}_r}{\partial \theta}\right|_{\mathbf{U}}\right) \tag{3}$$

This solution is repeated for each parameter in the vector $\mathbf{\Theta}$, reusing the factorization of $\mathbf{K}_T$. Full details of the DDM equation assembly and solution procedures are given in [11], including the recovery of other response derivatives from the nodal solution in Eq. (3).

3. Bridge girder modeling approach

In a general finite element setting, the most common approach to compute the moment and shear response of bridge girders is to subdivide each span into multiple elements with nodes corresponding to critical locations. Moving loads are taken into account as statically equivalent nodal forces and the bending moment and shear force at each critical location are determined from rigid body equilibrium at the element ends.

An alternative approach is taken in this study, where each span is considered as one force-based element [12] whose integration points coincide with critical locations. Using this integration approach, it is straightforward to link bending moment and shear forces to a constitutive model rather than relying on rigid body equilibrium [13]. Furthermore, moving loads are taken into account as part of the element, rather than nodal, equilibrium equations. The force-based formulation and its associated response sensitivity are described in the remainder of this section.

3.1. Force-based element formulation

Force-based beam elements are formulated in terms of vectors, $\mathbf{q} = [M_I \; M_J]^T$ and $\mathbf{v} = [\theta_I \; \theta_J]^T$, that represent the end moments and end rotations, respectively, of the beam, as shown in Fig. 1. At every section along the element, there is a bending moment and shear force, $\mathbf{s}(x) = [M(x) \; V(x)]^T$, and the corresponding curvature and shear deformation, $\mathbf{e}(x) = [\kappa(x) \; \gamma(x)]^T$. Without loss of generality, axial effects are omitted.

Equilibrium between section forces, basic forces, and moving loads is satisfied in strong form:

$$\mathbf{s}(x) = \mathbf{b}(x)\mathbf{q} + \mathbf{s}_p(x) \tag{4}$$

The matrix, $\mathbf{b}$, contains interpolation functions for the moment and shear forces along the beam.

$$\mathbf{b}(x) = \begin{bmatrix} x/L - 1 & x/L \\ 1/L & 1/L \end{bmatrix} \tag{5}$$

The vector, $\mathbf{s}_p$, in Eq. (4) describes the section forces due to member loads. For the case of a moving point load, this vector is described in terms of the location and magnitude of the load in the statically determinate basic system. Since moving loads are considered part of the element equilibrium equations in the force-based formulation, they are taken into account in $\mathbf{P}_r$ and $\partial \mathbf{P}_r / \partial \theta|_{\mathbf{U}}$ rather than $\mathbf{P}_f$ and $\partial \mathbf{P}_f / \partial \theta$ when assembling Eqs. (1) and (3), respectively.

Based on the principle of virtual forces, the element deformations, $\mathbf{v}$, are obtained in terms of section deformations, $\mathbf{e}$, along the element.

$$\mathbf{v} = \sum_{j=1}^{N_p} \mathbf{b}_j^T \mathbf{e}_j w_j \tag{6}$$

where $\mathbf{b}_j \equiv \mathbf{b}(x_j)$ and $\mathbf{e}_j \equiv \mathbf{e}(x_j)$ are the interpolation function and the deformation evaluated at the jth section along the element, with location, x_j, and integration weight, w_j.

The element flexibility matrix is obtained by linearization of Eq. (6) with respect to basic forces:

$$\mathbf{f} = \frac{\partial \mathbf{v}}{\partial \mathbf{q}} = \sum_{j=1}^{N_p} \mathbf{b}_j^T \mathbf{f}_{s_j} \mathbf{b}_j w_j \tag{7}$$

where $\mathbf{f}_s$ is the section flexibility matrix. The flexibility matrix in Eq. (7) is inverted to give the element stiffness matrix, $\mathbf{k} = \mathbf{f}^{-1}$, for subsequent assembly in the tangent stiffness matrix, $\mathbf{K}_T$, of Eq. (2). Full details of the force-based element implementation are given in [14].

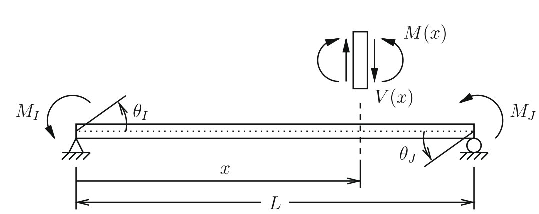

Fig. 1. Simply supported basic system for beam finite elements.