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Probabilistic fracture mechanics assessment of flaws in turbine disks including quality assurance procedures

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Abstract

A stochastic model for the failure of turbine disks including quality assurance procedures is established. The underlying reliability analysis is based on a fracture mechanics description using both a direct Monte Carlo simulation and a first-order reliability method. The failure probability and its sensitivity to input parameters are obtained together with confidence bounds with respect to uncertain input quantities. Assessment of the accuracy in probabilistic design is essential if only a limited amount of data is available. The results could be applied to extend the life in the respect that an inspection schedule can be derived from the calculated failure probabilities.

Keywords: Probabilistic fracture mechanics; Non-detection probability; Confidence bounds; Monte Carlo; FORM; Non-parametric bootstrap; Bayesian statistics

1. Introduction

Probabilistic fracture mechanics (PFM, for a survey of different fields see e.g. [20]) deals with the assessment of reliability or remaining lifetime of components containing real or postulated flaws in terms of probabilities attributed to a certain event of failure [10]. In this paper, it is assumed that other failure modes do not contribute to the failure probability, i.e. crack failure being the dominant failure mode.

At the core of each PFM analysis, a deterministic failure description based on either design codes or structural analysis is required. A PFM analysis directly evaluates failure probabilities P_f from the statistical uncertainties of material data, crack geometry, and loading. The distribution of flaw size changes with time if cyclic crack growth and non-destructive inspections and repair/removal of critical cracks are considered. For the present analysis, it is assumed that loading and operating conditions have low variability and the resulting stresses can be taken as deterministic as obtained by a finite element analysis.

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For the evaluation of P_f, two different numerical algorithms were applied, a direct Monte Carlo simulation (MC) and the first-order reliability method (FORM). While MC is straightforward to use but requires a large amount of computing efforts especially for small failure probabilities, FORM provides quick estimates for P_f and sensitivities with respect to the input variables but does not provide any error estimate of the result, so that results have to be checked, e.g., by MC reference calculations.

Fundamentals of the reliability analysis together with the PFM model are given in Section 2. The PFM procedure is applied to assess the reliability of a turbine disk in a Siemens gas turbine. Details of the underlying FM model are given in Section 2.2. Confidence bounds can be determined by applying non-parametric bootstrap methods to the input data. The methods are outlined in Section 2.6. Some details of the quality assurance (QA) procedure are presented in Section 3. Section 4 deals with the description of the available data in terms of statistical distributions. Results for P_f including sensitivities and confidence intervals are given in Section 5, where also the relation between probabilistic and deterministic FM approaches is addressed.

Apart from the specific example presented in this paper, the method presented has a much wider scope. Its application covers all those structural components in mechanical as well as in civil engineering, where continuous monitoring and inspection is necessary and where uncertainty assessment enhances confidence in the results of reliability analyses.

2. Theory

In structural reliability methods, a deterministic description is necessary for component failure assessment, while a statistical analysis is required to calculate the failure probability from the scatter of the random input quantities. This section describes the underlying theory of the fracture mechanics based failure function as well as of the computational methods for the failure probability assessment.

2.1. Fundamentals of reliability analysis

Given basic random input variables $\mathbf{X} = (X_1,\ldots,X_N)$ with respective probability density functions $f_{X_i}(X_i)$, the failure of the component is described by a failure function $g(\mathbf{X})$ and the failure probability P_f is then defined by the multi-dimensional integral

$$P_f = \int_{g(X)\leqslant 0} dX_1 \ldots dX_N f_{X_1}(X_1)\ldots f_{X_N}(X_N), \tag{1}$$

where the integration has to be carried out over the failure domain $g(\mathbf{X}) \leqslant 0$. Here, for the sake of simplicity, the $\mathbf{X}$ are assumed to be stochastically independent [1,2].

It is sometimes convenient to quantify the amount of safety for a structure or component by the reliability index β which is related to P_f via

$$P_f = \Phi(-\beta) \tag{2}$$

with the cumulative standard normal distribution function Φ. A typical failure probability value of $P_f = 10^{-6}$ corresponds to a reliability index of $\beta = 4.768$.

Before presenting algorithms to solve Eq. (1), the underlying fracture mechanics (FM) failure model is reviewed.

2.2. The g-function and the FM model

In the PFM approach, candidates for the basic random variables $\mathbf{X}$ are loads, flaw geometry (size, shape, location of the flaw), and material data (fracture toughness, yield and ultimate tensile strength). For the following problem, the g-function of Eq. (1) is taken as the R6 Rev 04 Approximate Option 2 failure assessment curve for continuous yielding [3]