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Model of observed stochastic balance between work and free time supporting the LQTAI definition

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Abstract

A balance differential equation between free time and money-producing work time on the national economy level is formulated in a previous paper in terms of two dimensionless quantities, the fraction of work time and the total productivity factor defined as the ratio of the Gross Domestic Product to the total salary paid in return for work. Among the solutions there is one relation that compares surprisingly well with the relevant sequences of Danish data spanning from 1948 to 2003, and also with similar data from several other countries except for slightly different model parameter values. Statistical analysis reveals a data structure that allows the formulation of a simple stochastic model for the development of the data sequences with the year. Simulations with the stochastic model show sample curve behavior of the same fluctuating appearance as the factual data. This indicates that there are no other significant systematically balance influencing parameters on the macro economical level than those considered in the definition in the previous paper of the Life Quality Time Allocation Index.

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1. Introduction

The Life Quality Time Allocation Index defined in [1] was derived on the basis of a few assumptions of axiomatic nature and a simple principle of equilibrium between production value equivalent work time and free time. Surprisingly these assumptions were subsequently supported strongly by official joint economical data, population data, and work time data from Denmark in the time span from 1948 to 2003. Support is constituted also through corresponding data from several other countries, even though the span of years of registered data is noticeably shorter than for Denmark [2].

The investigation reported in this paper serves the purpose of demonstrating that the deviations from the smooth theoretical curves in Fig. 1 of [1] (reprinted below as Fig. 1) may be of purely random nature. In fact,

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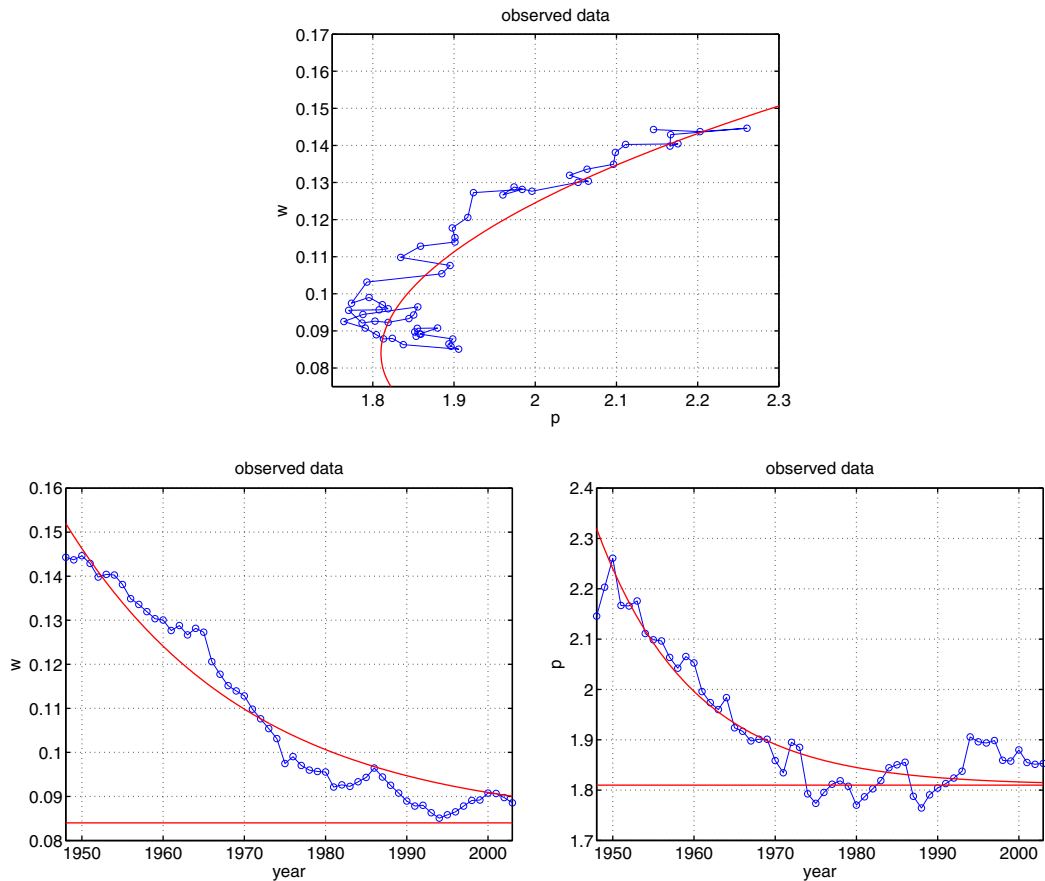*E-mail address:* oveditlevsen@mac.com

Fig. 1. (Top) Data points of (p, w) for Denmark in the years 1948–2003. The smooth curve is the relation (3) for $K = 1.81c(1 - c)^{-(1-1/c)}$ and $c = 0.084$. (Bottom left and right) Recorded work time fraction w and time equivalent productivity (ratio of salary to GDP per same time unit) p for Denmark from 1948 to 2003. The smooth curves are jointly least square fits of (4) (left) and (3) with (4) substituted (right) and are consistent with the curve in the top diagram.

the statistical analysis of the data series reveals a simple stochastic structure that is easily modeled in terms of autoregressive sequences. Simulations with the model give sample sequences that in their trends and fluctuations show great similarity with the data sequences.

The main result for the definition of the Life Quality Time Allocation Index in [1] is that the balance weight between production value enhanced work time and free time is a constant that for Denmark can be estimated from the above mentioned data. For the completeness the derivation from [1] is repeated here in a simpler way.

2. Balance equation

To appreciate the balance principle formulated below, it is essential to accept the assumption that any amount of time used for some purpose has a value to the user depending on his or her activity during the considered time. The value concept of interest herein should be anticipated as an average over the population. Consumed time is classified into two categories only, that is, into the category of money-producing work time and the category of free time for leisure and other non-paid free time activity.

An increment of the work time fraction of the total time has a monetary value directly proportional to the monetary value of the production in the time increment, of course. However, what herein is called "experienced worth" is reasonably defined as the dimensionless ratio of this increment value to the value of the production in the entire work time fraction of the total time. This definition of experienced worth of work time is